School of Electrical and Information Engineering

University of the Witwatersrand, Johannesburg

ELEN3016 Control I

Test

(Closed-book; One double-sided A4 formula sheet allowed.)

Time: 90 minutes **Total: 40** **Date: 21 September 2011**

Question 1 [20 marks]

The mechanical assembly shown in Figure 1 consists of two masses M_1 and M_2 with interconnecting springs and dampers. For the sake of our analysis ignore gravity (i.e. set $g = 0 \text{ m/s}^2$). The input to the system is $y_1(t)$, the position of mass M_1, and the output of the system is $y_2(t)$, the position of mass M_2.

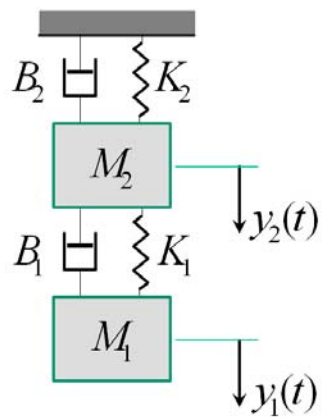

Figure 1: Mechanical assembly.

a. Derive the transfer function relating $y_2(t)$ (as output) to $y_1(t)$ (as input). (10)

b. Draw the state diagram of the system. (6)

c. Verify the transfer function derived in (a.) by applying block diagram transformation theorems to the state diagram derived in (b.). (4)

Question 2 [20 marks]

A helicopter stabilisation system is shown in Figure 2(a).

a. Construct the root locus for this system. (10)

b. Determine the gain values K at break-in and/or breakaway points. (4)

c. Determine the gain K for which the system will be marginally stable as well the frequency of oscillation during marginal stability. (6)

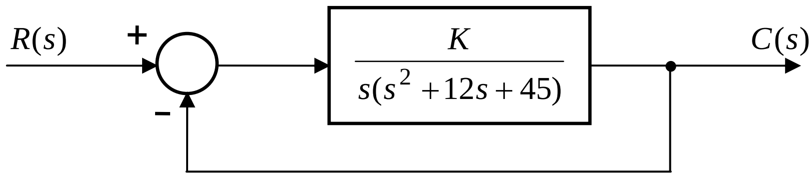

Figure 2. Helicopter stabilisation system.

Table of Laplace Transforms

	$f(t) = \mathscr{L}^{-1}[F(s)]$	$F(s) = \mathscr{L}[f(t)]$
1.	$\delta(t)$	1
2.	$t^n u(t)$, $n = 0,1,2,\ldots$	$\frac{n!}{s^{n+1}}$, $n = 0,1,2,\ldots$
3.	$t^n e^{\lambda t} u(t)$, $n = 0,1,2,\ldots$	$\frac{n!}{(s-\lambda)^{n+1}}$, $n = 0,1,2,\ldots$
4.	$e^{\alpha t} \sin\beta t\, u(t)$	$\frac{\beta}{(s-\alpha)^2+\beta^2}$
5.	$e^{\alpha t} \cos\beta t\, u(t)$	$\frac{s-\alpha}{(s-\alpha)^2+\beta^2}$
6.	$\frac{1}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$

Important Properties of the Laplace Transform

Operation	**Time domain** $f(t)$	**Frequency domain** $F(s)$
Scaling $(a > 0)$	$f(at)$	$\frac{1}{a} F\left(\frac{1}{a}\right)$
Time shift $(t_o > 0)$	$f(t-t_o)u(t-t_o)$	$F(s)e^{-st_o}$
Time differentiation	$\frac{df}{dt}(t)$	$sF(s) - f(0^-)$
Time integration	$\int_{-\infty}^{t} f(t)\, dt$	$s^{-1}F(s) + s^{-1}\int_{-\infty}^{0} f(t)\, dt$
Frequency shift	$f(t)e^{s_o t}$	$F(s-s_o)$