ELEN3016 Control Engineering Laboratory

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1 Introduction

The inverted pendulum is probably one of the most famous of the mechanical balancing-type problems. The objective of this laboratory is to understand this balancing problem at a fundamental level and develop a control algorithm which regulates the pendulum in the upright position. The laboratory follows the usual approach to model based control problems. Firstly, you need to model the pendulum, using suitable assumptions. Next, the open-loop system must be simulated in either Matlab, Octave, Mathematica or Maxima. This is the model validation phase. Next, the control algorithm must be designed and tested on the software model. Lastly, the control algorithms designed will be demonstrated by the tutors on the physical rotary inverted pendulum.

2 Model Development

Modelling of the rotary inverted pendulum system from fundamental physics is difficult. The complete system involves a DC-motor which actuates the arm, a rotary encoder for the arm joint and a rotary encoder for the pendulum joint. A sketch of the system is supplied in Figure 1. The arm joint rotates in the xy plane and the pendulum joint rotates in the zy plane when the arm is extended along the x direction.

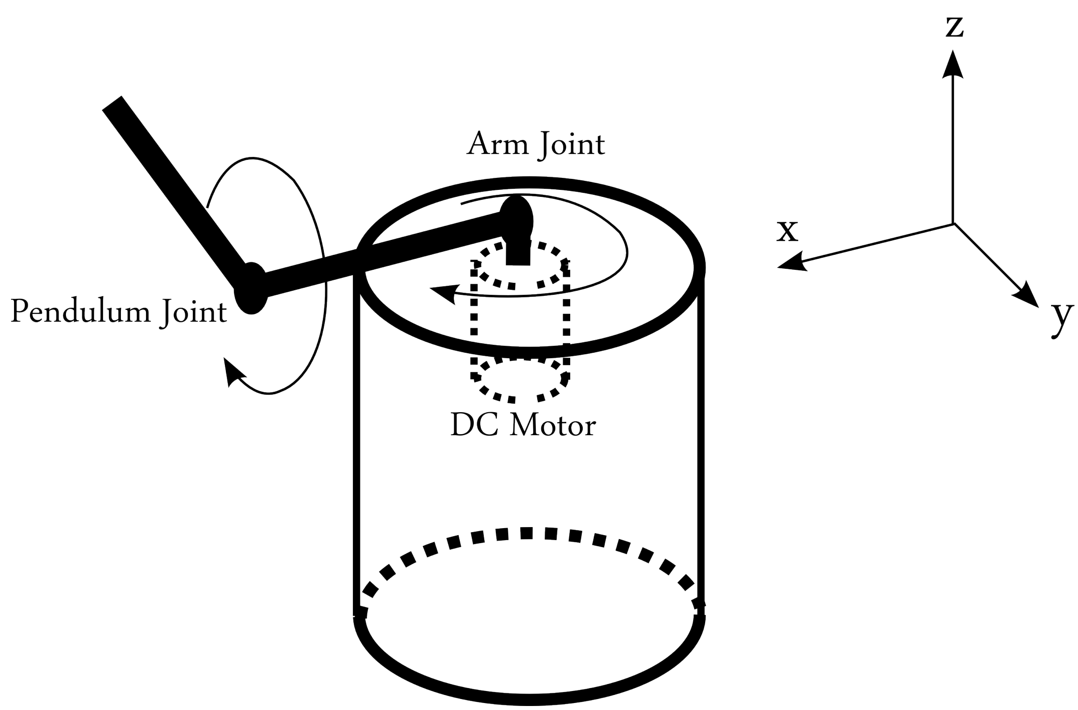

Figure 1: Sketch of Rotary Inverted Pendulum

If the above description confuses you, don't worry. We'll approximate the model using the free-body diagram in Figure 2.

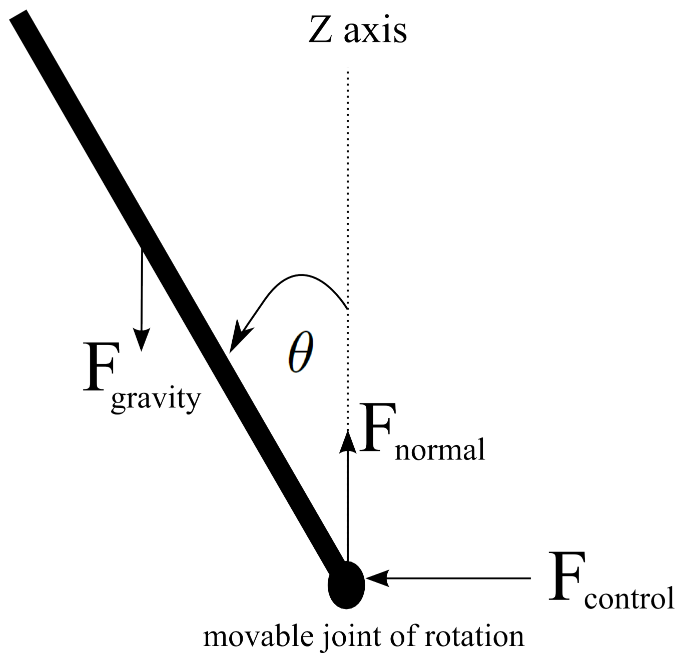

Figure 2: Free-Body Approximation

It is recommended that you take torques about the center of mass of the pendulum. You can model F_{control} using the arm length, l_a and the torque from the DC motor, τ.
The DC motor also has a gear box, with a gear ratio of 30 : 1.
Now,

$$F_{\text{control}} = \frac{\tau}{l_a}.$$

The voltage to the DC motor is the input, u. The angle of the pendulum is θ. **Note: there is viscous friction present in the pendulum joint**.
Hence, the torque due to friction is

$$\tau_f = -B_f\dot{\theta}.$$

There may also be noise present on the measured output. The block diagram of the system is depicted in Figure 3.

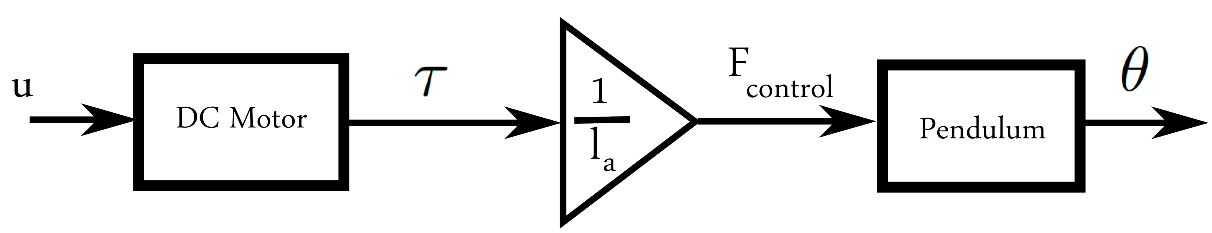

Figure 3: Block Diagram of Pendulum System

The model that you derive should depend heavily on θ. It is a good idea to use the small angle approximation and hence,

$$\begin{aligned}\cos(\theta) &\approx 1 \\ \sin(\theta) &\approx \theta.\end{aligned}$$

This is a linearised model of the pendulum system.

3 Parameters of Rotary Inverted Pendulum

The parameters of the system are presented in Table 1.

Table 1: Parameters of System

Parameter Name	**Symbol**	**Value**	**Units**
Arm Length	l_a	0.083	metres
Pendulum Length	l_p	0.205	metres
Pendulum Mass	m_p	0.031	kilograms
Motor Torque Constant	K_m	0.0273	Newton metre/Ampere
Motor DC Resistance	R_m	3.5	Ohms
DC Motor Gear Ratio	N	30 : 1	-
Viscous Friction Co-efficient	B_f	5×10^{-4}	Newton metre second/radian

4 Software Model Verification

Using only low level code, simulate the open-loop model derived previously. You can code a fancy integration algorithm or use the one presented next. Here is an example of a Backwards Euler approximation to integration. Given a differential equation, with output y and input u,

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = f(y, u),$$

from the fundamental theorem of calculus,

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \approx \frac{y(kT) - y(kT - T)}{T},$$

where T is the sampling period and k is the sample number.
Hence,

$$\begin{aligned}\frac{y(kT) - y(kT - T)}{T} &= f\left(y(kT), u(kT)\right) \\ y(kT) &= y(kT - T) + T\, f\left(y(kT), u(kT)\right).\end{aligned}$$

This can be written more succinctly as,

$$y[k] = y[k-1] + T\, f\left(y[k], u[k]\right).$$

This is an algorithm that can be coded iteratively with a **for** loop.

4.1 Questions

1. Is the linear model stable or unstable? Explain why.
2. Can the system output ever climb to infinity? Is this physically realistic?
3. Make your initial angle such that the pendulum is perfectly in the upright position. What happens? Why?
4. Make your initial angle slightly larger than the perfect upright position. What happens? Why?
5. What happens when the angle is near the hanging down position? Is this physically realistic?

4.2 Bonus Marks

Simulate the system without making any small angle approximations.

1. Can the system output ever climb to infinity? Is this physically realistic?
2. Make your initial angle slightly larger than the perfectly upright position. Compare this plot to the previous case.
3. What happens when the angle is near the hanging down position? Is this physically realistic?

5 Control Design

Using the small-angle approximations, design a Lead controller and a Proportional-Derivative (PD) controller for your system. Derivatives add noise into the system so cascade a unity gain low pass filter onto the derivative term.
For the Lead controller, try to achieve a gain margin $>$ -15 dB and a phase margin $> 40°$.
For the PD controller, try to achieve a damping ratio of 0.707 with as low a natural frequency as possible.
To test the controller, look at the disturbance impulse response. This is a regulation problem and you need to regulate the system about the upright position.

5.1 Questions

1. What is the difference between the Lead controller and the PD controller? Why not use a PID controller?
2. Can you make the gains arbitrarily small and still regulate the angle about the upright position?
3. What is the maximum gain that you can apply and still regulate about the upright position? Is this realistic?
4. Monitor the input voltage, what happens to it as you increase the gains?

5.2 Bonus Marks

Test your controllers on the original non-linear system.

1. Do your controllers still work when the initial angle is close to upright?
2. How large can you make the initial angle and still regulate about the upright position?
3. What happens when the initial condition is near the hanging condition?

6 Hardware Demonstration

A time will be set aside later in the course to demonstrate the controllers that students have designed.

7 Laboratory Procedure

The "D-lab" will be made available for you to use in order to complete the laboratory questions. The laboratory is to be done in groups of two. A verbal examination will be conducted. There will be times available that groups can book to be examined. These times are binding.