CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 20)

Overview

- First Things First!
- Gain and Phase Margins
- 1st-Order Controllers
- Phase-Lead Compensator Design Methods
- Phase-Lead Compensator Example
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

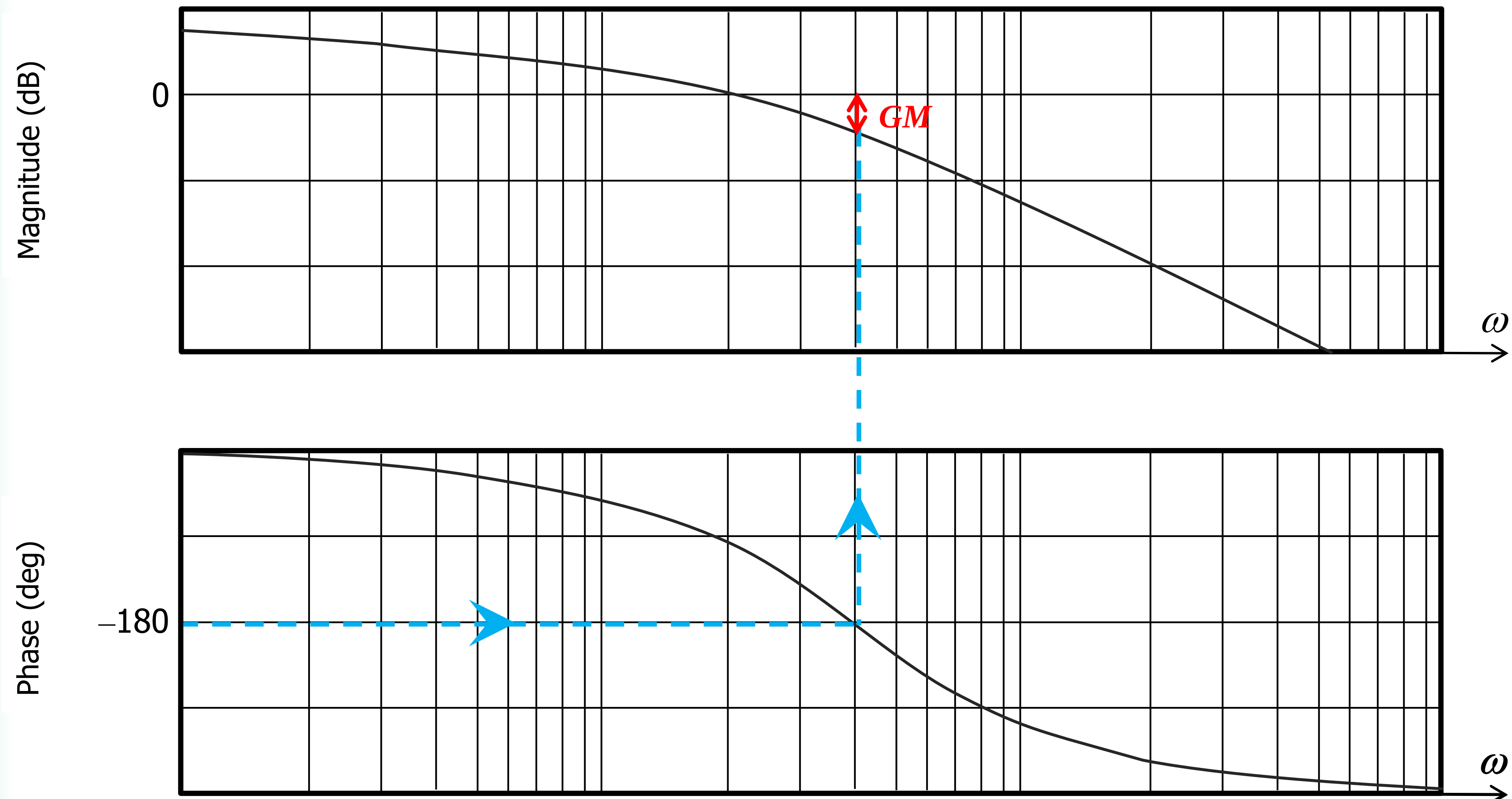
- None

Gain and Phase Margins

- Gain & Phase Margins Explained
  - *Gain margin* is the amount of gain increase that the system can tolerate before it becomes unstable.
  - *Phase margin* is the amount of time delay that the system can tolerate before it becomes unstable.
  - Gain & phase margins are just different ways of looking at same thing – system stability.
  - If the gain margin is zero then so is the phase margin and vice versa. Thus they are in total agreement about the onset of marginal stability.

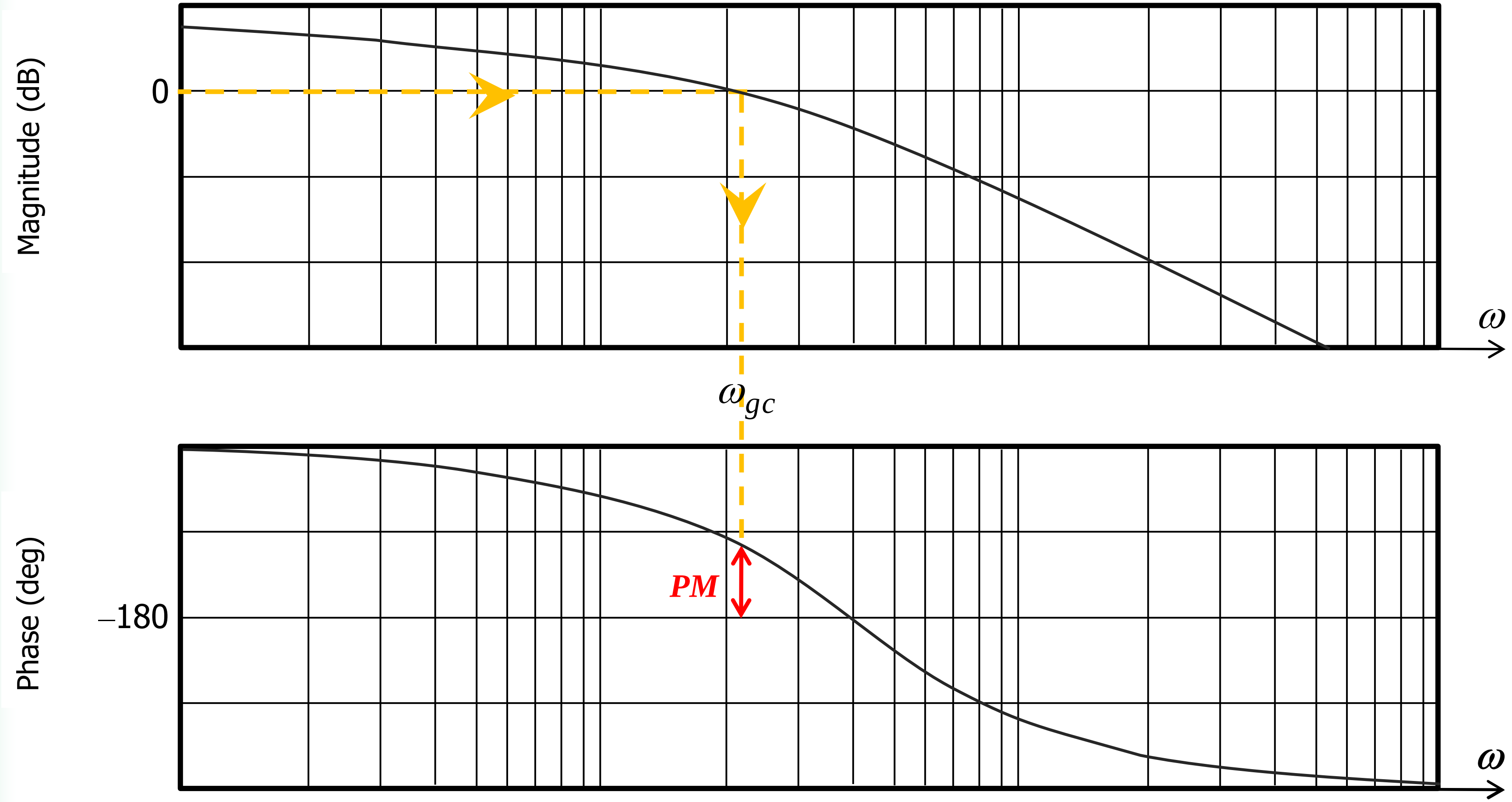
Gain and Phase Margins

Gain Margin on the Bode plot of $G(j\omega)H(j\omega)$.



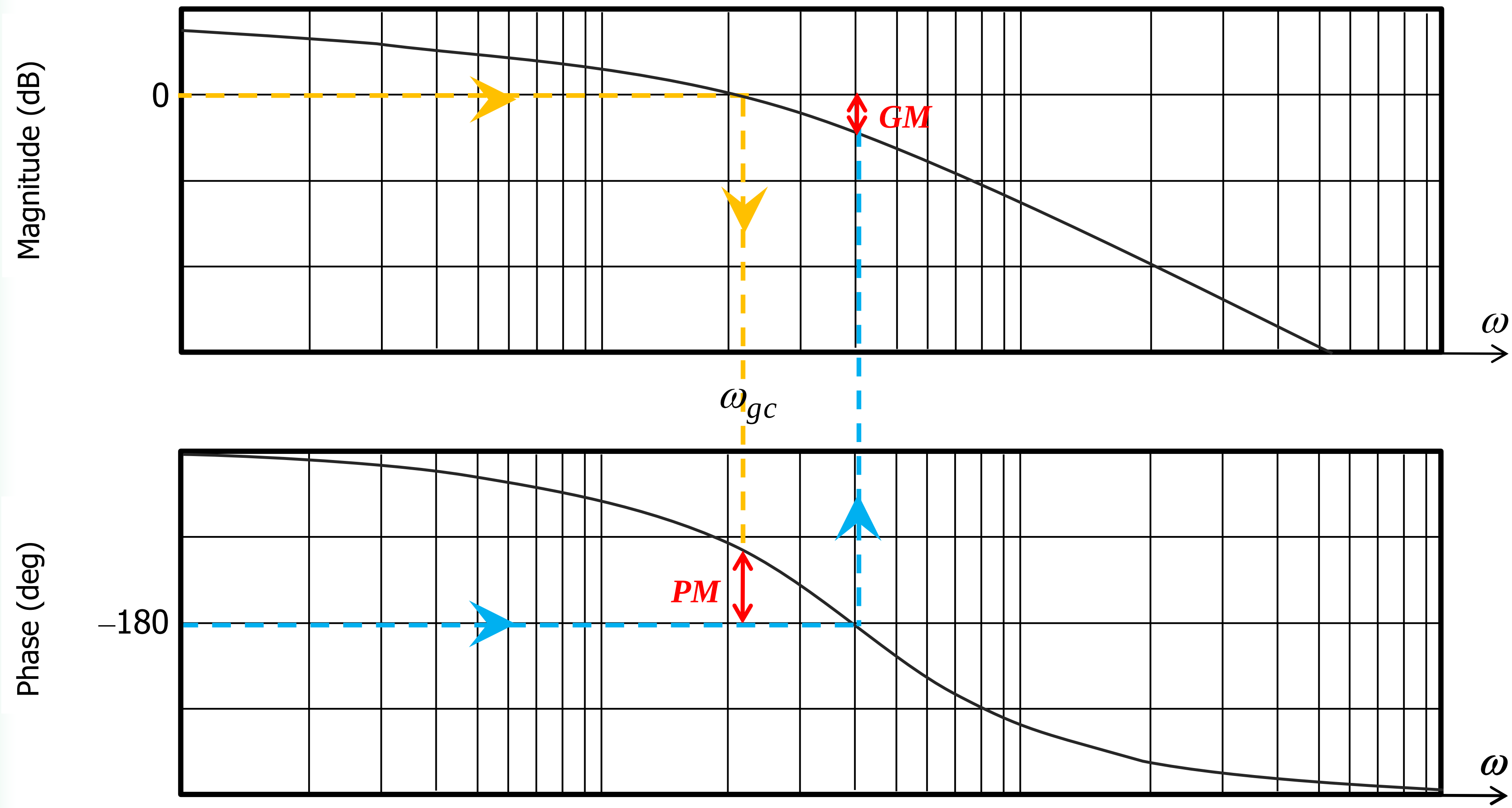
Gain and Phase Margins

Phase Margin on the Bode plot of $G(j\omega)H(j\omega)$.



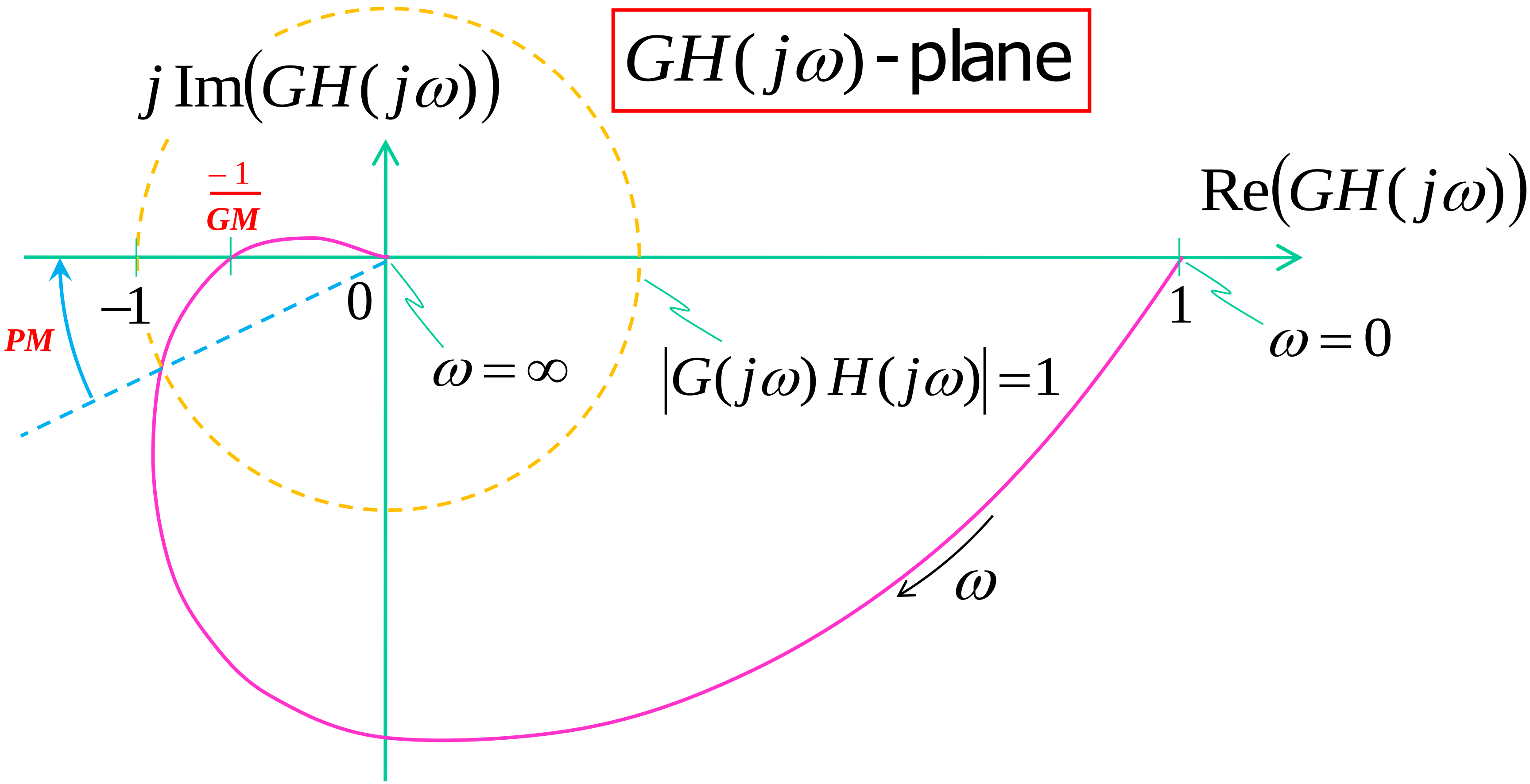
Gain and Phase Margins

Gain and Phase Margins on the Bode plot of $G(j\omega)H(j\omega)$.



Gain and Phase Margins

Gain and Phase Margins on the Nyquist plot of $G(j\omega)H(j\omega)$.

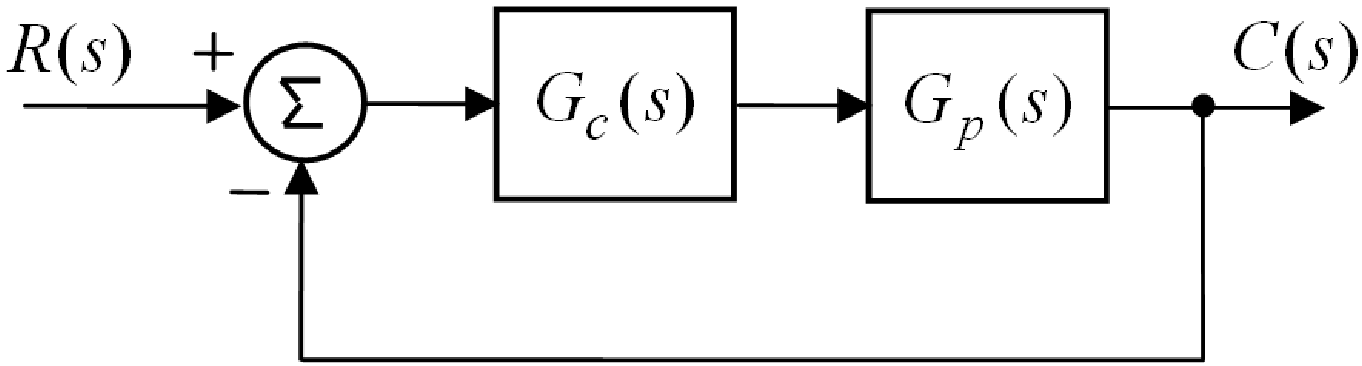


1st-Order Controllers

- Different Controller-Plant Configurations
  - Controller in the forward path – *cascade* controller
  - Controller in the feedback path – *feedback* controller
- Controller Formulations
  - Rational Function of Polynomials (RFoP)
  - Time Constant (TC)

1^{st}-Order Controllers

- Generic 1^{st}-Order (Cascade) Controller
  - Controller-system configuration considered:



Controller: $G_c(s)$, a 1^{st}-order system

Plant: $G_p(s)$, arbitrary.

1^{st}-Order Controllers

- 1^{st}-Order Controller – RFoP Formulation
  - 1^{st}-order controller:

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}, \qquad a_0, a_1, b_1 > 0$$

DC gain: a_0

Zero: $z = -a_0/a_1$

Pole: $p = -1/b_1$

$$\alpha = \frac{p}{z} = \frac{a_1}{a_0 b_1}$$

1st-Order Controllers

- 1st-Order Controller – TC Formulation
  - 1st-order controller:

$$G_c(s) = K_c \frac{\tau_z s + 1}{\tau_p s + 1}, \quad \tau_z, \tau_p, K_c > 0$$

DC gain: K_c

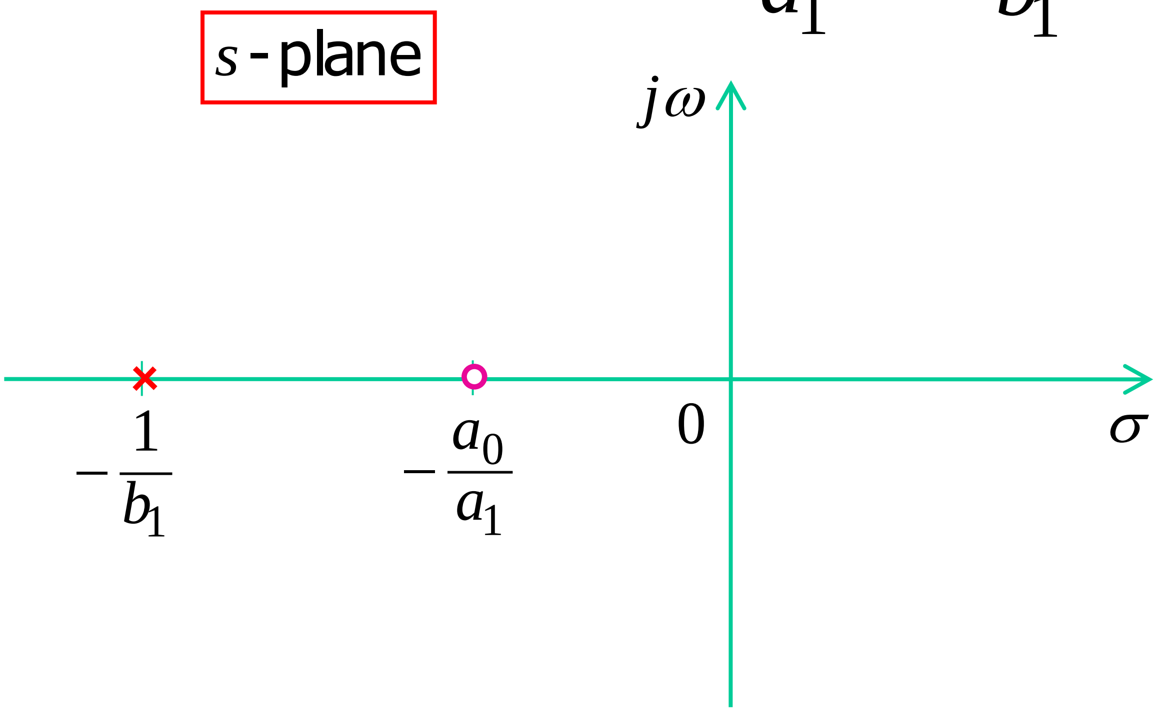
Zero: $z = -1/\tau_z$

Pole: $p = -1/\tau_p$

$$\alpha = \frac{p}{z} = \frac{\tau_z}{\tau_p}$$

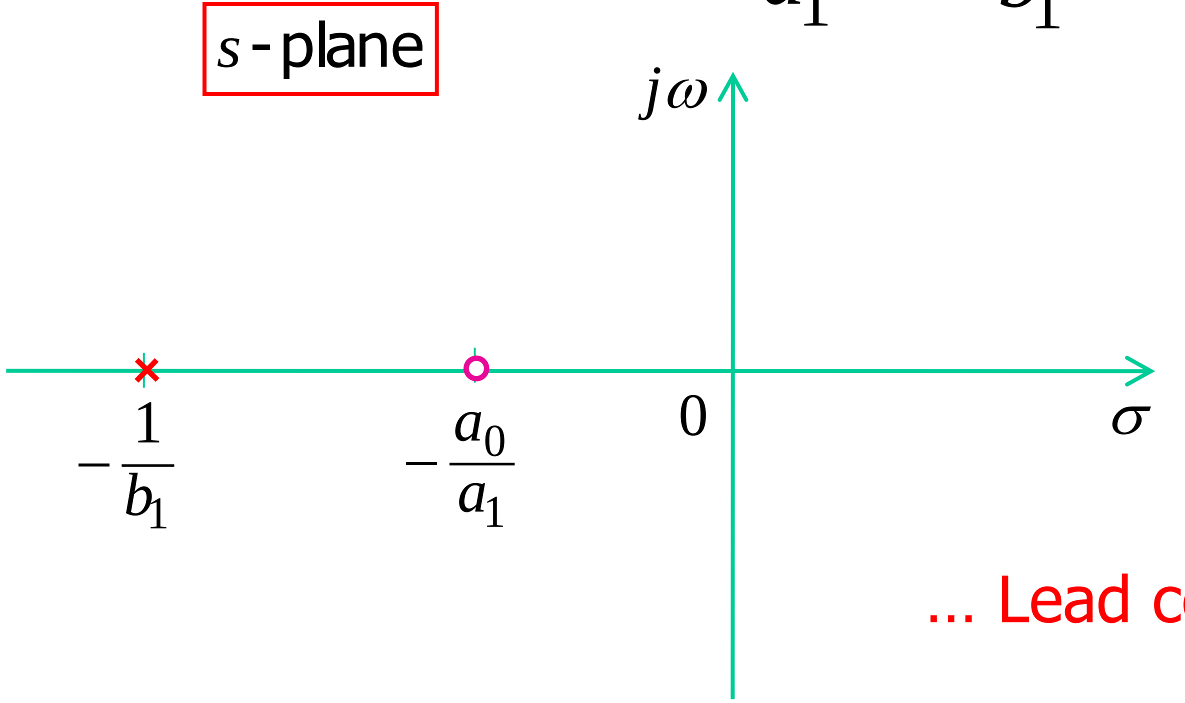
1st-Order Controllers

- Approximate PD Controller
  - Requirement: $z = -\dfrac{a_0}{a_1} < -\dfrac{1}{b_1} = p$



1^{st}-Order Controllers

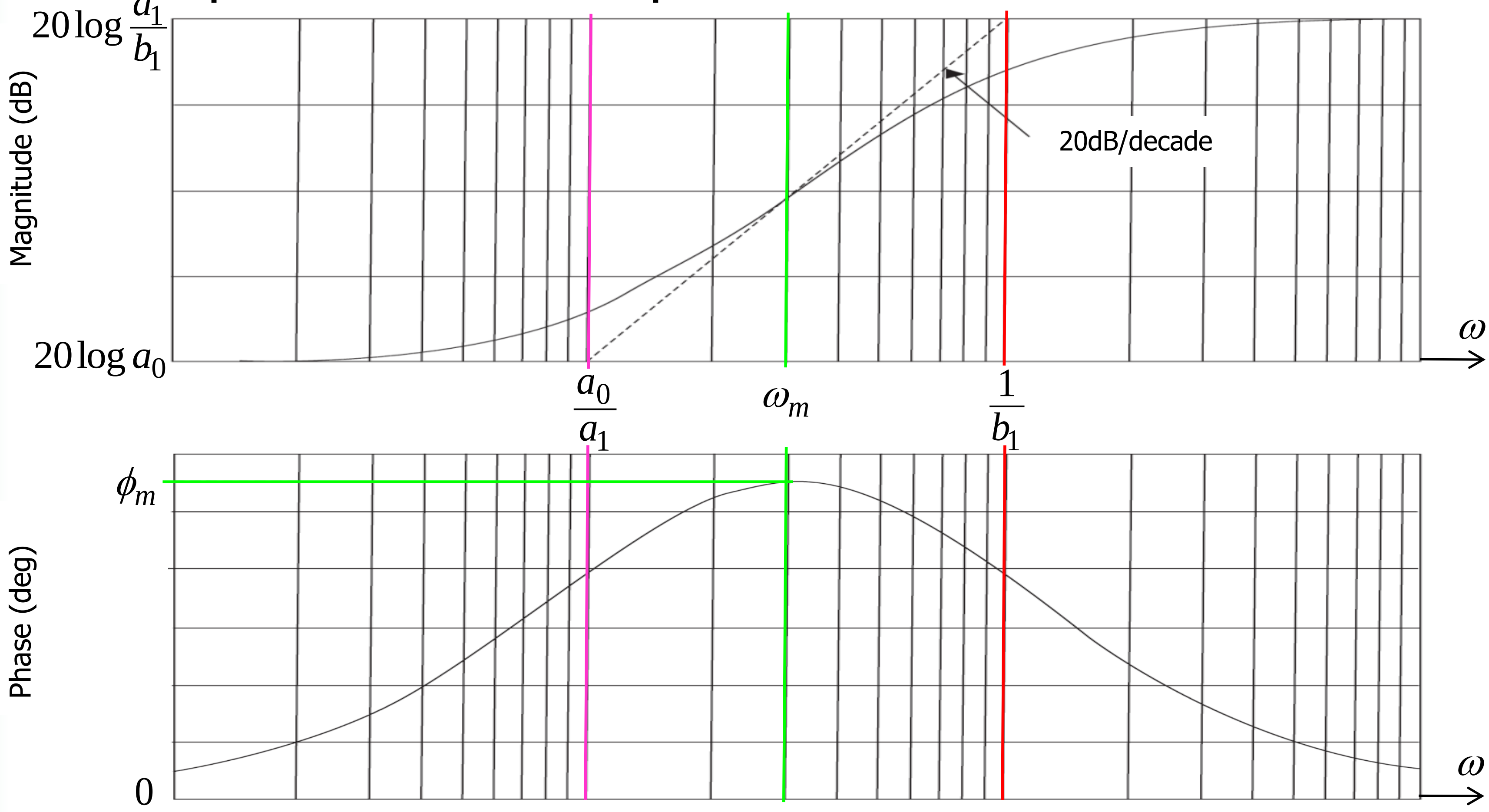
- Approximate PD Controller
  - Requirement: $z = -\dfrac{a_0}{a_1} < -\dfrac{1}{b_1} = p$



… Lead compensator

Lead Compensator

Bode plots for lead compensator



Lead Compensator

- Analysis of the Phase-Lead Compensator
  - Frequency at maximum phase advance is

$$\omega_m = \sqrt{\frac{a_0}{a_1 b_1}}$$

  - Corresponding maximum phase advance is

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1} \quad \text{or} \quad \alpha = \frac{1+\sin\phi_m}{1-\sin\phi_m}$$

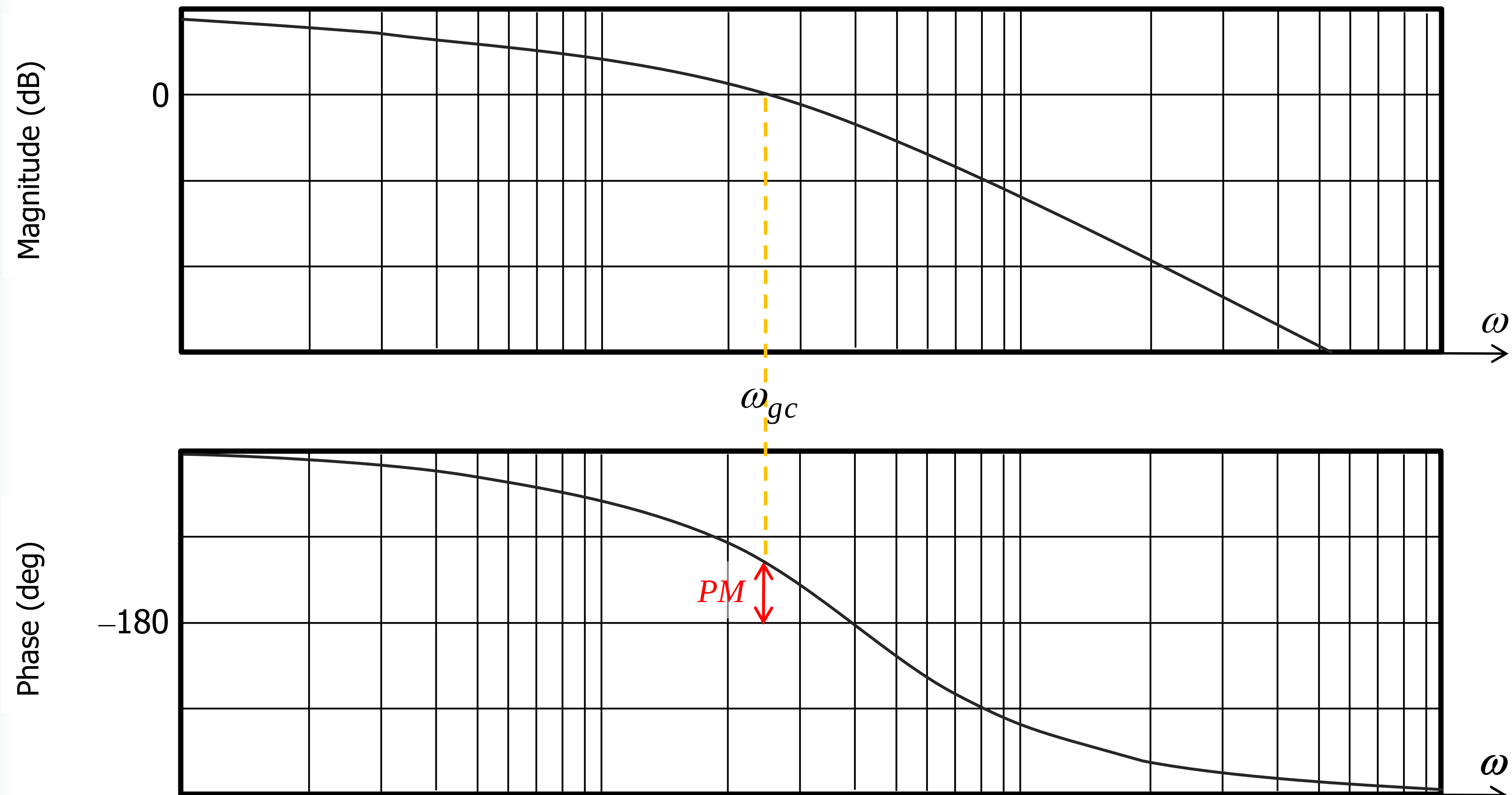
Lead Compensator

- Lead Compensator Design – Method 1
  - Select a_0 to meet any specified steady-state error spec.
  - Plot the *uncompensated* open-loop frequency response to obtain the phase margin, PM, and modulus crossover frequency, ω_{gc}, defined by $|G_p(j\omega_{gc})| = 1$.
  - Set ω_m equal to ω_{gc} and estimate the phase advance needed to ensure required phase margin for the closed-loop system.
  - Plotting the *compensated* open-loop frequency response reveals that ω_{gc} has increased. Reducing the compensator gain then returns ω_{gc} to its original value.

  Study the examples in Burns.

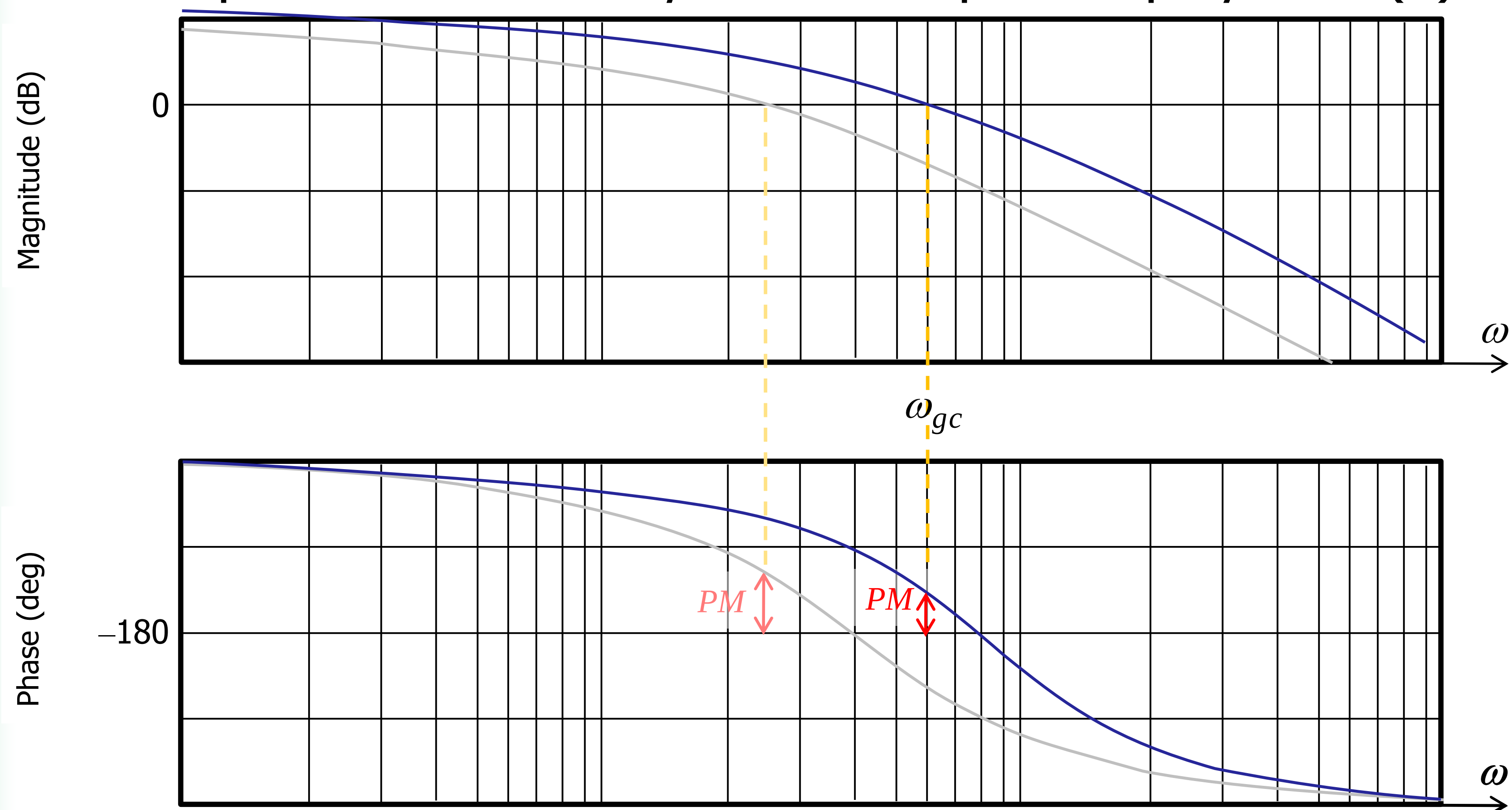
Lead Compensator

Bode plots for *uncompensated* open-loop system



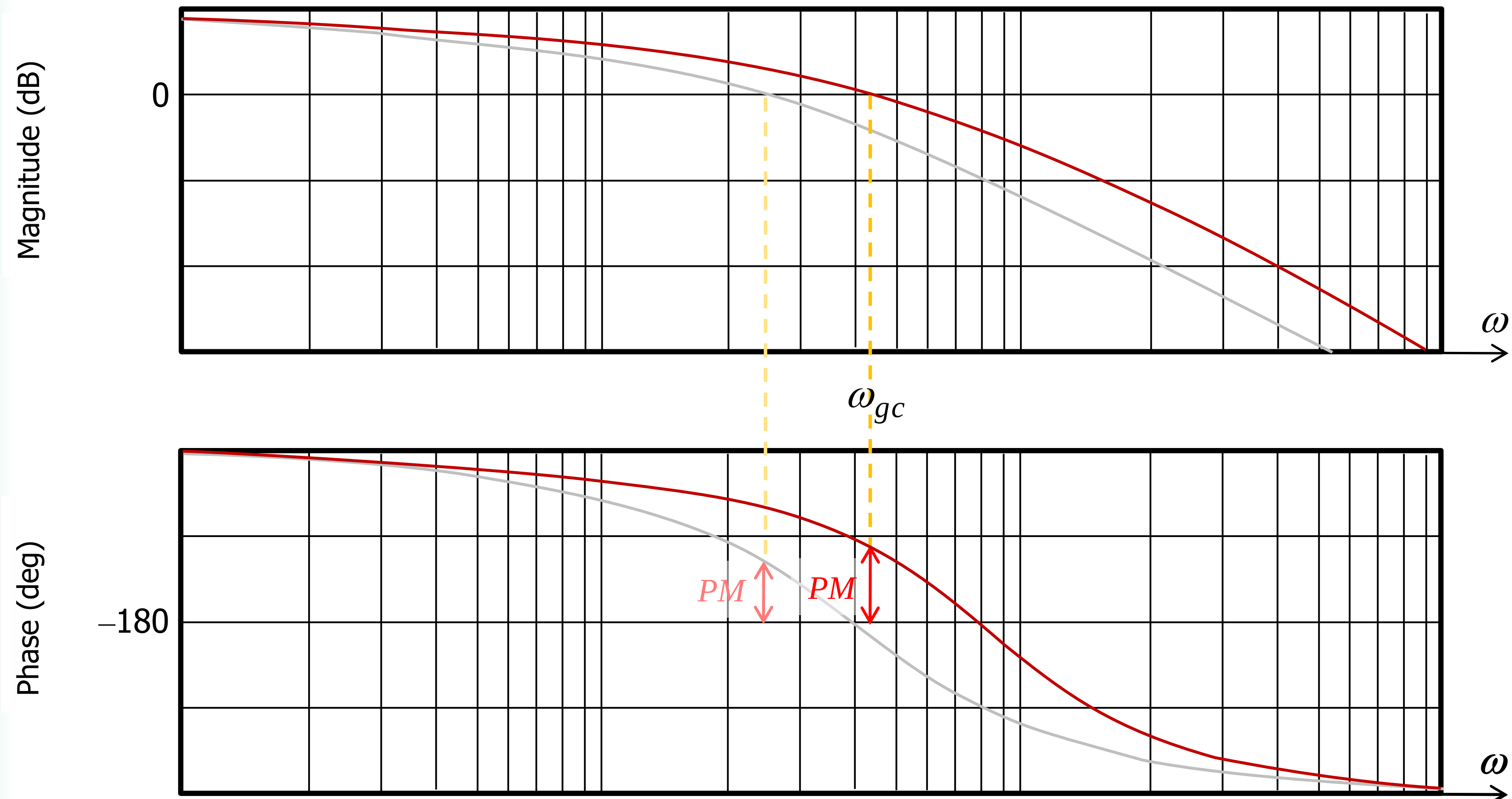
Lead Compensator

Bode plots for lead-*compensated* open-loop system (1)



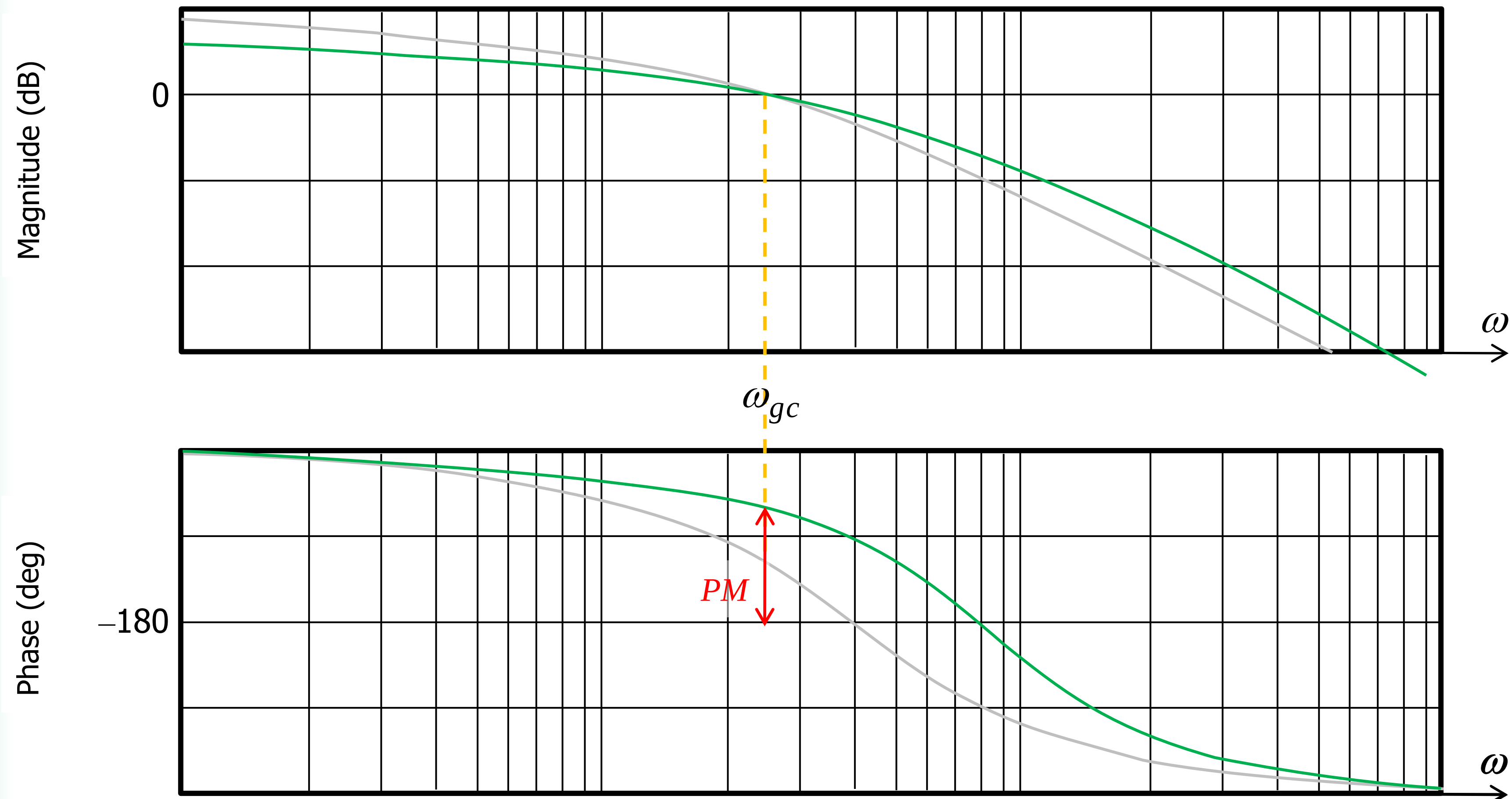
Lead Compensator

Bode plots for lead-*compensated* open-loop system (2)



Lead Compensator

Bode plots for lead-*compensated* open-loop system (3)



Lead Compensator

- Lead Compensator Design – Method 2
  - Select a_0 to meet any specified steady-state error spec.
  - Plot the frequency response for $a_0 G_p(j\omega)$ to obtain the phase margin, PM_{actual}.
  - Estimate the phase advance needed to ensure required phase margin for the compensated system, and associated value of α:

$$\phi_m = PM - PM_{\text{actual}} + 5^\circ \qquad \alpha = \frac{1+\sin\phi_m}{1-\sin\phi_m}$$

  - Choose ω_m as the frequency at which the magnitude of the uncompensated system passes through the value $-10\log\alpha$ (dB). (Due to the gain of $-10\log\alpha$ added at ω_m by the compensator, ω_m will become the new modulus crossover frequency.)

Lead Compensator

- Lead Compensator Design – Method 2
  - Plot the compensated open-loop frequency response and determine its phase margin.
  - If the required phase margin is achieved stop. If not, increase the value of α. To avoid the controller starting to act like a differentiator ensure that $\alpha \leq 10$. If an α-value of greater than 10 is required, introduce a second lead compensator in series with the first lead compensator.
  - Close the loop and determine appropriate closed-loop responses (i.e. transient response and frequency response).

Study the examples in Burns and in Raven. (Omit this year)

Lead Compensator

- Lead Compensator Design – Method 3
  - Select a_0 to meet any specified steady-state error spec.
  - Plot the frequency response of $a_0 G_p(j\omega)$.
  - Given the modulus-crossover frequency ω_{gc} as well as phase margin PM required, the compensated open-loop system must satisfy

$$G_c(j\omega_{gc})G_p(j\omega_{gc}) = \frac{a_1 j\omega_{gc} + a_0}{b_1 j\omega_{gc} + 1} \underbrace{M_G e^{j\theta_G}}_{G_p(j\omega_{gc})} = 1 e^{j(-180^\circ + PM)}$$

giving the following design equations,

$$a_1 = \frac{1 + a_0 M_G \cos(PM - \theta_G)}{-\omega_{gc} M_G \sin(PM - \theta_G)} \qquad b_1 = \frac{\cos(PM - \theta_G) + a_0 M_G}{\omega_{gc} \sin(PM - \theta_G)}$$

after some algebra.

Lead Compensator

- Method 3 cont'd
  - Draw compensated open-loop Bode plots and inspect design.
  - Close the loop and determine appropriate closed-loop responses (i.e. transient response and frequency response).
  - If the required closed-loop system's performance is not met, reduce the phase margin specified and repeat the design to obtain the controller $G_{c,1}(s)$. Afterwards design an additional lead controller $G_{c,2}(s)$ for the remaining phase margin. For the design of the additional lead controller, the plant is now considered to be $G_{c,1}(s)\,G_p(s)$.

Lead Compensator

- Example
  - Plant, $G_p(s) = \dfrac{400}{s(s^2+30s+200)}$.
  - Specifications:

    Unit ramp steady-state error: 10%

    Crossover frequency: $\omega_{gc} = 14$ rad/s

    Phase margin: $PM = 45^\circ$
  - Open-loop poles: $s = 0, -10, -20$.

Lead Compensator

- Example cont'd
  - Ramp error constant:

$$K_v = \lim_{s \to 0} s\, G_c(s)\, G_p(s) = \lim_{s \to 0} \frac{400\, a_0}{(s^2 + 30s + 200)} = 2a_0$$

Lead Compensator

- Example cont'd
  - Ramp error constant:

$$K_v = \lim_{s\to 0} s\,G_c(s)\,G_p(s) = \lim_{s\to 0} \frac{400\,a_0}{(s^2+30s+200)} = 2a_0$$

  - Unit ramp steady-state error:

$$e_{ss}\Big|_{\substack{\text{Unit}\\\text{Ramp}}} = \frac{R}{K_v}\Big|_{R=1} = \frac{1}{2\,a_0} = 10\%$$

$$\Rightarrow \quad a_0 = \frac{1}{2\times 0.1} = \frac{10}{2} = 5\ (=14\text{dB})$$

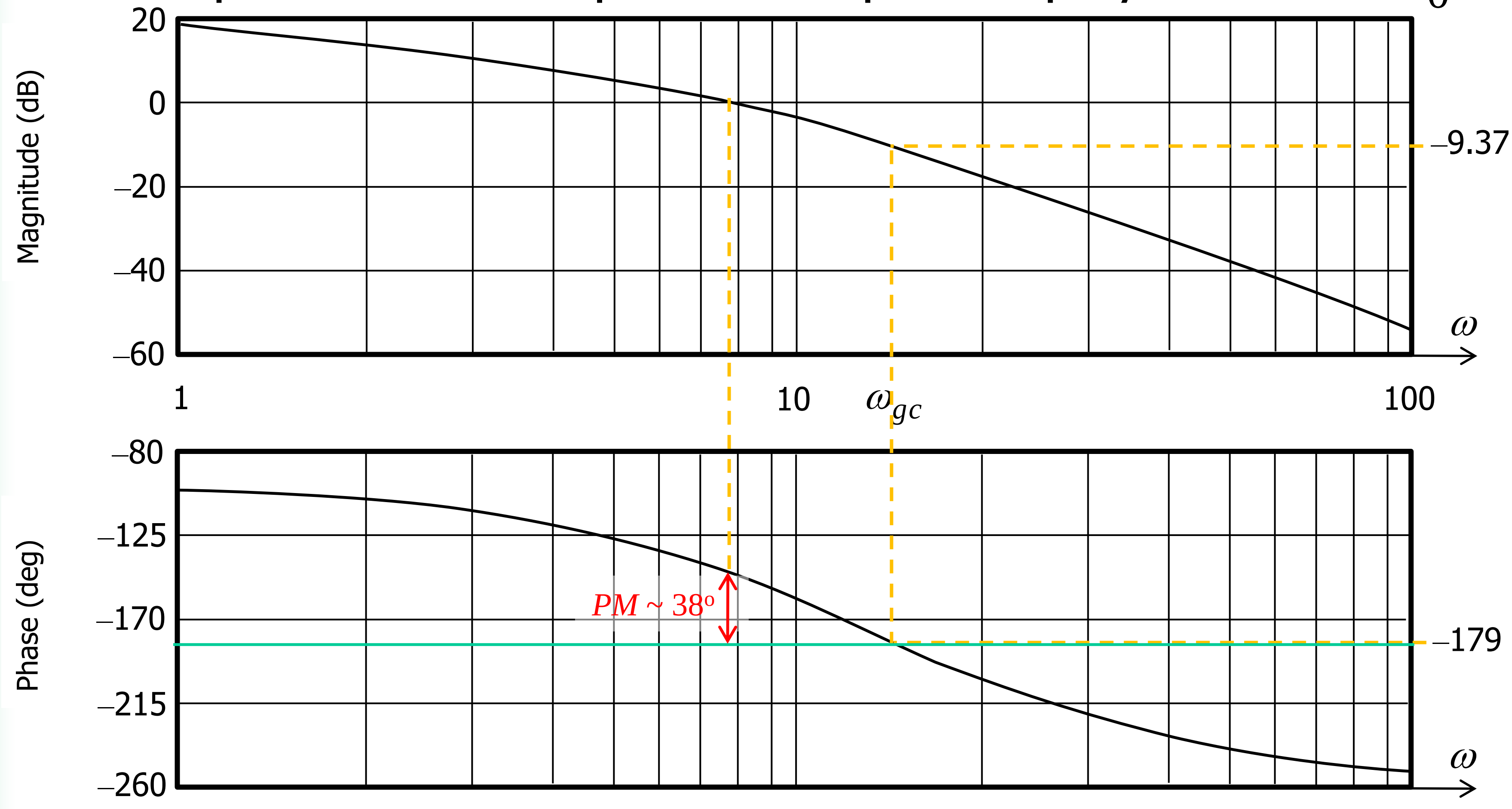
(Additional gain needed)

Lead Compensator

- Example cont'd
  - To satisfy steady-state error select $a_0 = 5$.
  - Next, draw Bode plots for $a_0 G_p(j\omega)$.
  - At $\omega = \omega_{gc} = 14$ rad/s we find $a_0 M_G = 0.34$ and $\theta_G = -179^\circ$.
  - The above design equations yield $a_1 = 1.1423$ and $b_1 = 0.0390$.
  - Controller/compensator: $G_c(s) = 5\dfrac{0.229s+1}{0.039s+1}$

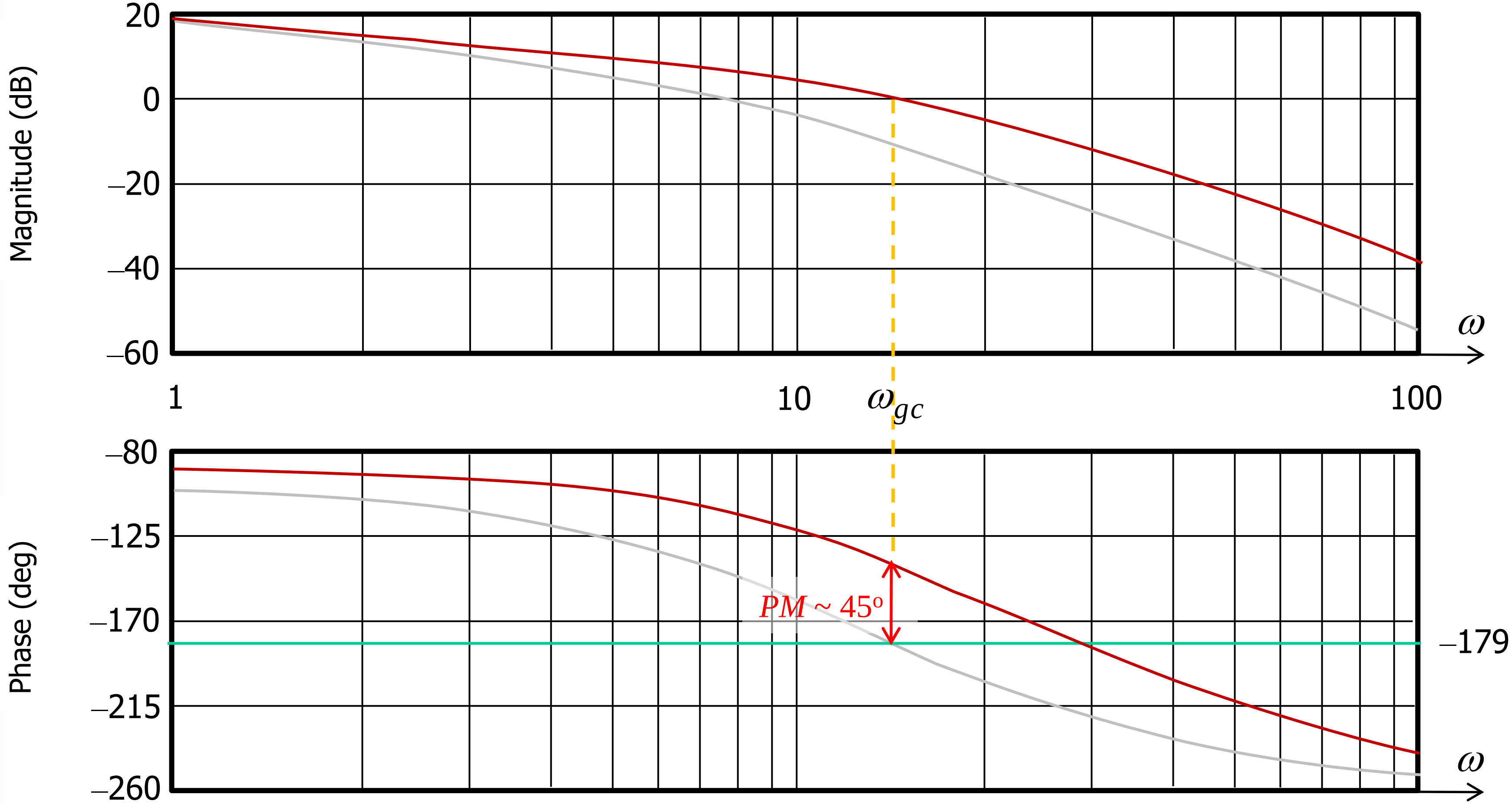
Lead Compensator

Bode plots for uncompensated open-loop system with a_0



Lead Compensator

Bode plots for compensated open-loop system



Tutorial Exercises & Homework

- Tutorial Exercise
  - Derive the formulas for the analytical phase-lead compensator design (i.e. Method 3).
- Homework
  - Study all relevant sections in Burns.

Conclusion

- Gain and Phase Margins
- 1^{st}-Order Controllers
- Phase-Lead Compensator Design Methods
- Phase-Lead Compensator Example
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Phase-Lag Compensators
  (Burns, Chapter 6)

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Thank you!

Any Questions?