CONTROL I

ELEN3016

Closed-Loop Control Systems

(Lecture 6)

Overview

- First Things First!
- Closed-Loop Control Systems
- Examples
- Tutorial Exercises & Homework

- Next Attraction!

Closed-Loop Systems

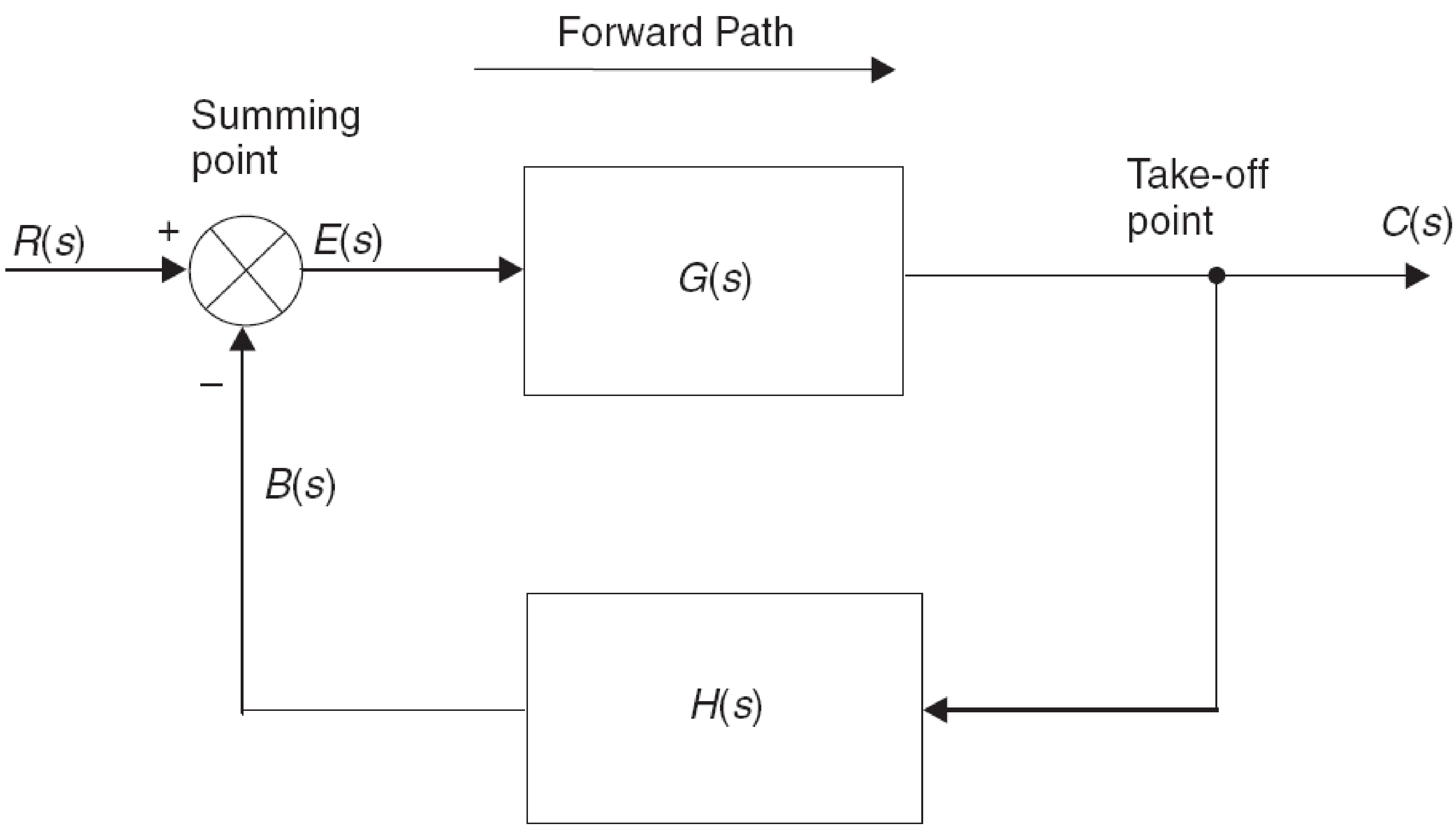


Figure 4.1 (Burns)

Closed-Loop Systems

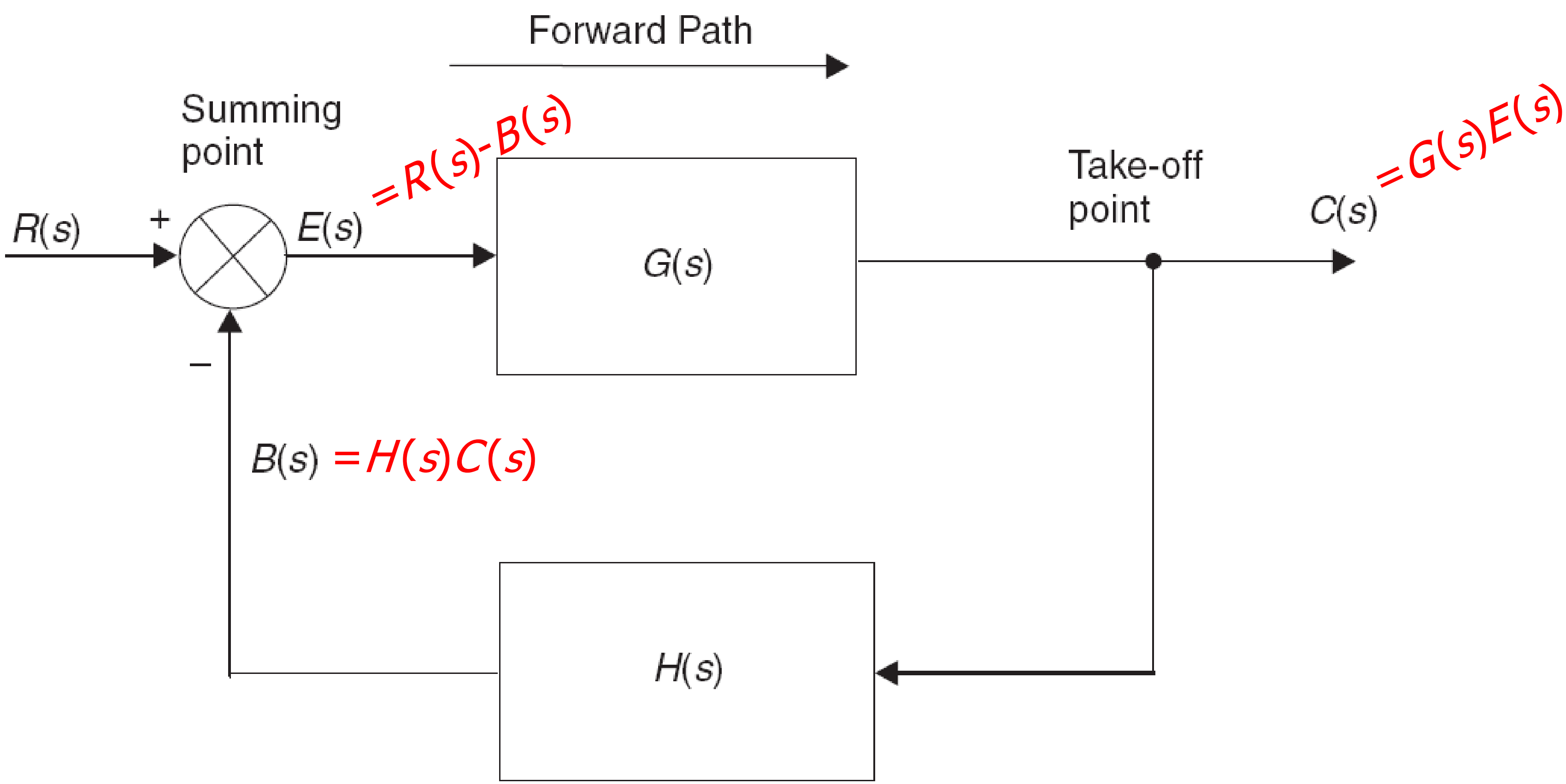


Figure 4.1 (Burns)

Closed-Loop Systems

From Figure 4.1

$$C(s) = G(s)E(s) \tag{4.1}$$

$$B(s) = H(s)C(s) \tag{4.2}$$

$$E(s) = R(s) - B(s) \tag{4.3}$$

Closed-Loop Systems

From Figure 4.1

$$C(s) = G(s)E(s) \tag{4.1}$$

$$B(s) = H(s)C(s) \tag{4.2}$$

$$E(s) = R(s) - B(s) \tag{4.3}$$

How do we proceed from here?

Closed-Loop Systems

From Figure 4.1

Eliminate!

$$C(s) = G(s)E(s) \tag{4.1}$$

$$B(s) = H(s)C(s) \tag{4.2}$$

$$E(s) = R(s) - B(s) \tag{4.3}$$

Substituting (4.2) and (4.3) into (4.1)

$$C(s) = G(s)\{R(s) - H(s)C(s)\}$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

$$C(s)\{1 + G(s)H(s)\} = G(s)R(s)$$

$$\frac{C}{R}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{4.4}$$

Closed-Loop Systems

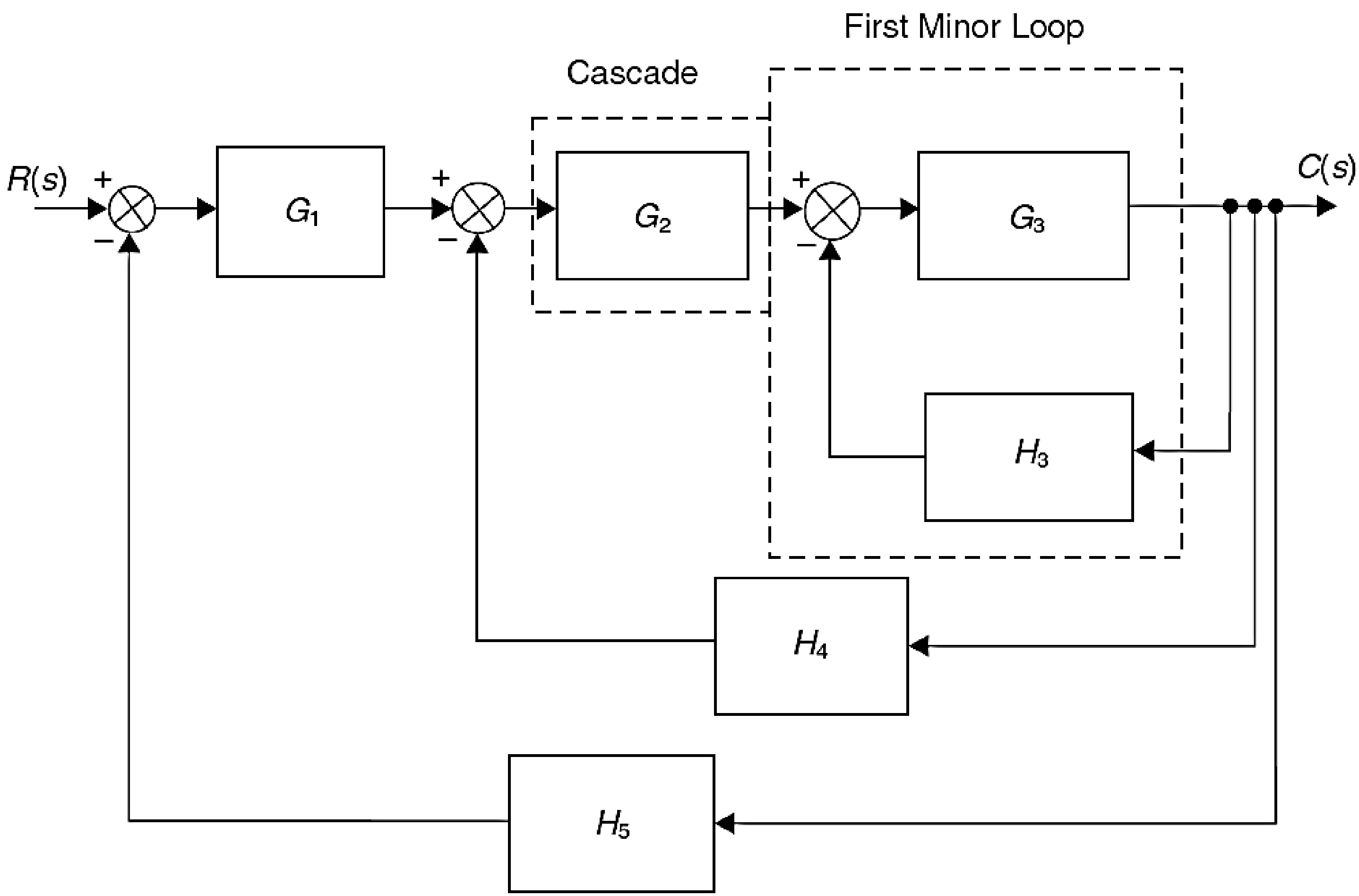


Figure 4.2 (Burns)

Closed-Loop Systems

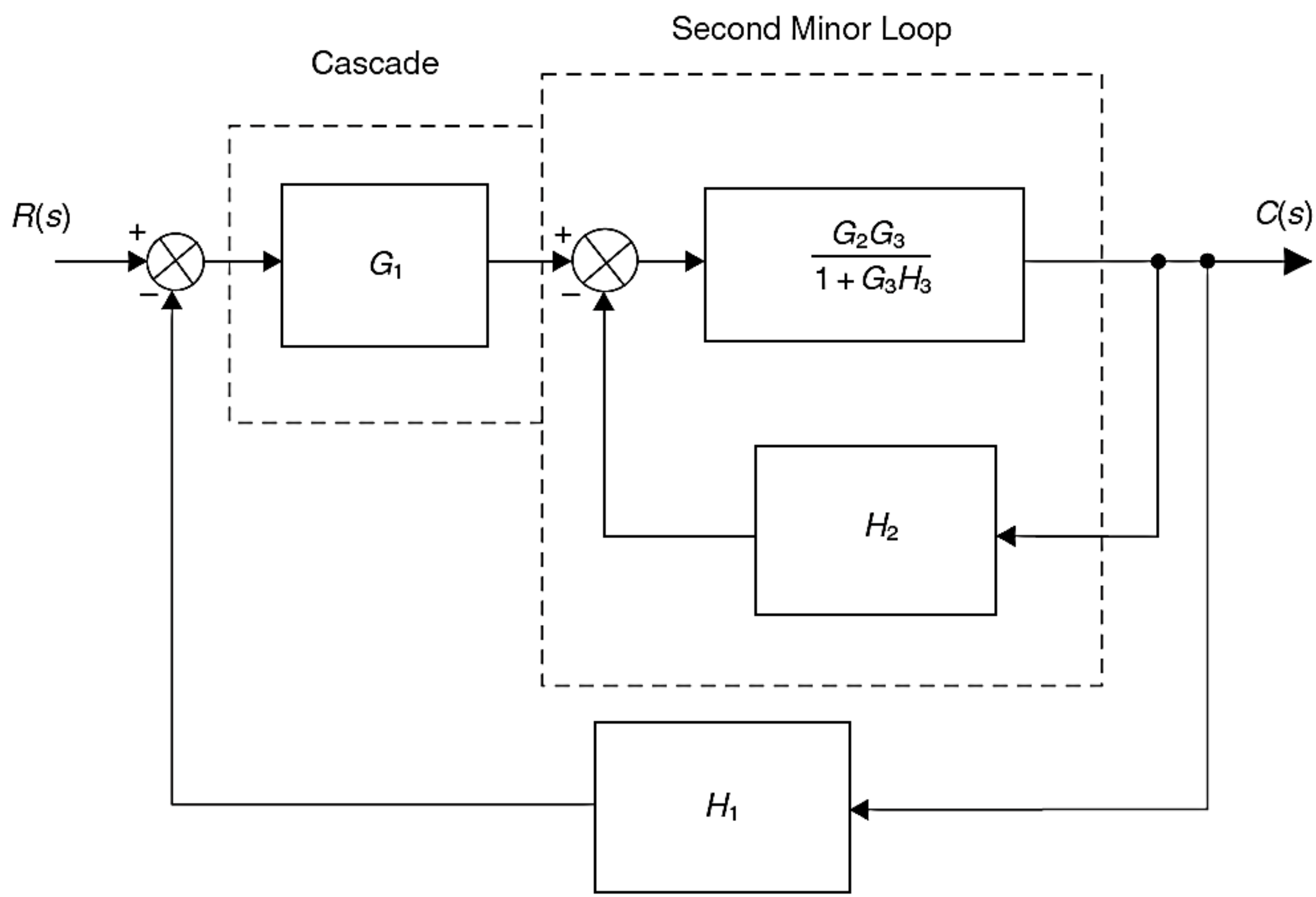


Figure 4.3 (Burns)

Closed-Loop Systems

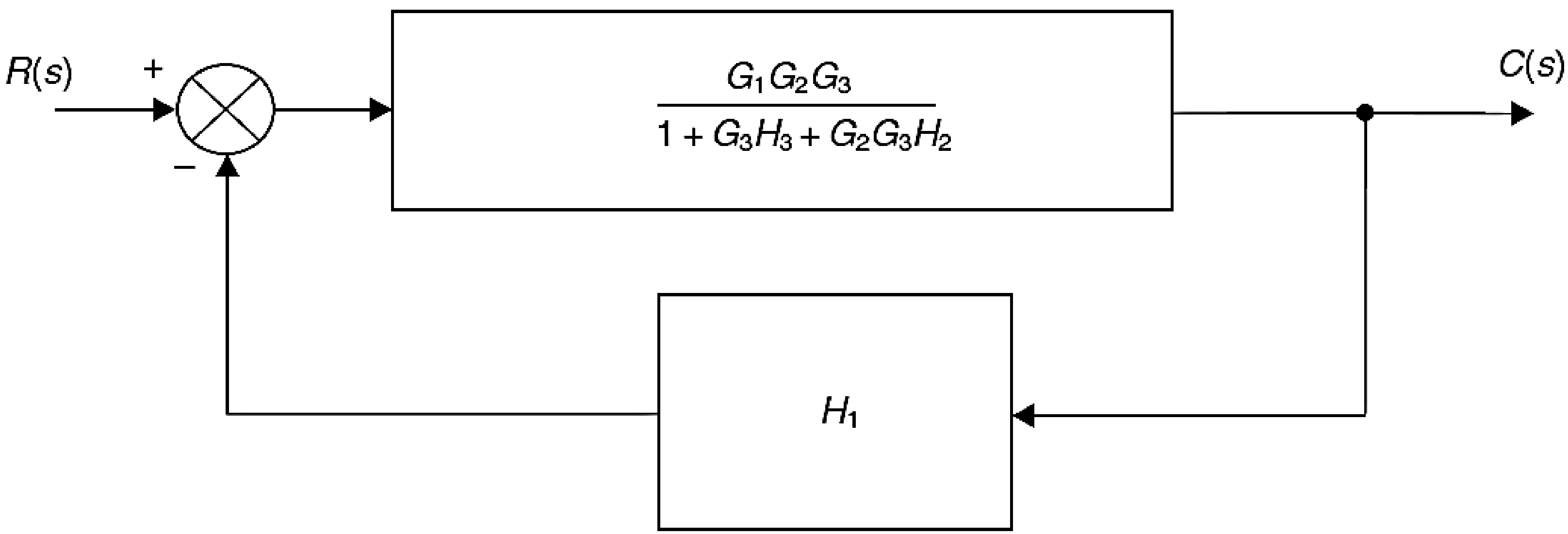


Figure 4.4 (Burns)

Closed-Loop Systems

$$\frac{C}{R}(s) = \frac{\frac{G_1 G_2 G_3}{1+G_3 H_3+G_2 G_3 H_2}}{1 + \frac{G_1 G_2 G_3 H_1}{1+G_3 H_3+G_2 G_3 H_2}}$$

Always simplify to avoid being penalised.

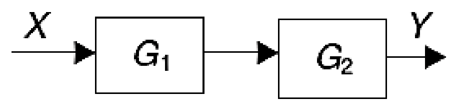
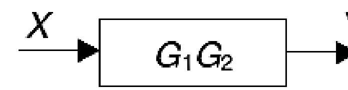
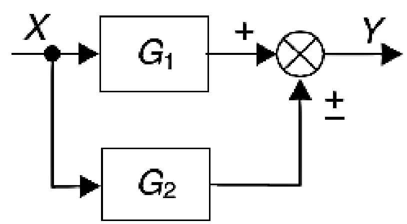
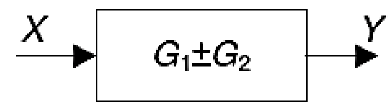
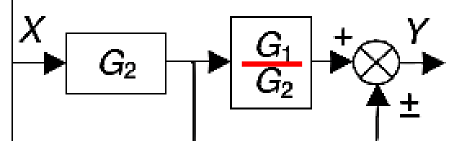
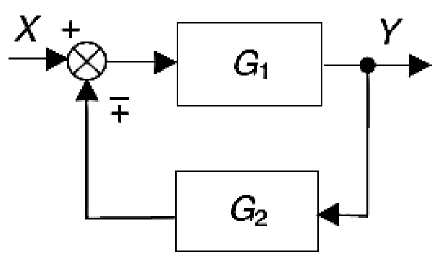
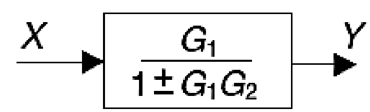
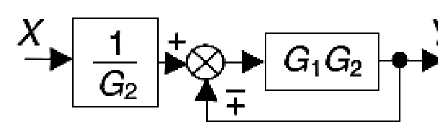
$$\frac{C}{R}(s) = \frac{G_1(s)G_2(s)G_3(s)}{1 + G_3(s)H_3(s) + G_2(s)G_3(s)H_2(s) + G_1(s)G_2(s)G_3(s)H_1(s)}$$

Closed-Loop Systems

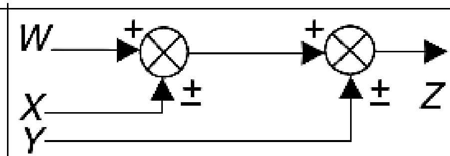
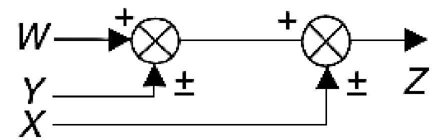
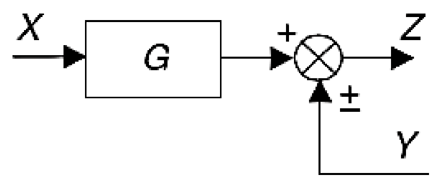
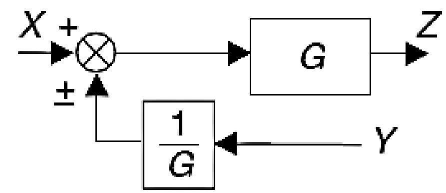
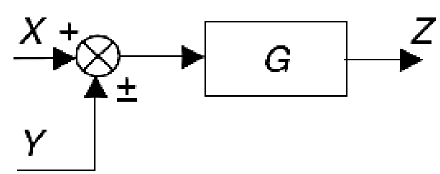
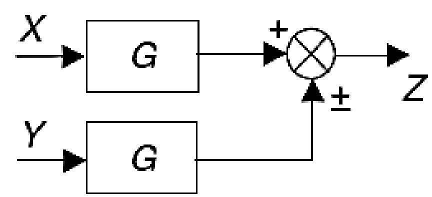
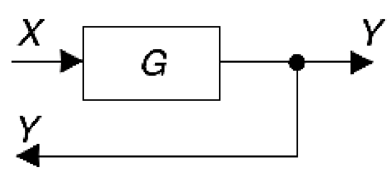
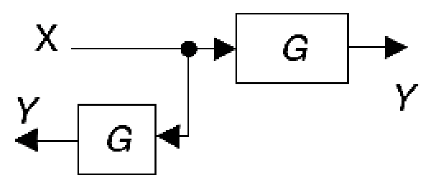
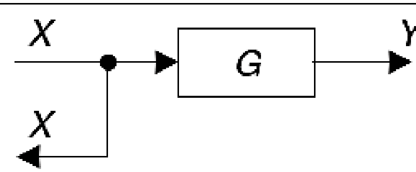
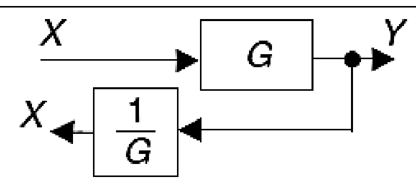
$$\frac{C}{R}(s) = \frac{\frac{G_1 G_2 G_3}{1+G_3 H_3 + G_2 G_3 H_2} \times (1+G_3 H_3 + G_2 G_3 H_2)}{\left(1 + \frac{G_1 G_2 G_3 H_1}{1+G_3 H_3 + G_2 G_3 H_2}\right) \times (1+G_3 H_3 + G_2 G_3 H_2)}$$

$$\frac{C}{R}(s) = \frac{G_1(s)G_2(s)G_3(s)}{1 + G_3(s)H_3(s) + G_2(s)G_3(s)H_2(s) + G_1(s)G_2(s)G_3(s)H_1(s)}$$

Block Diagram Manipulation

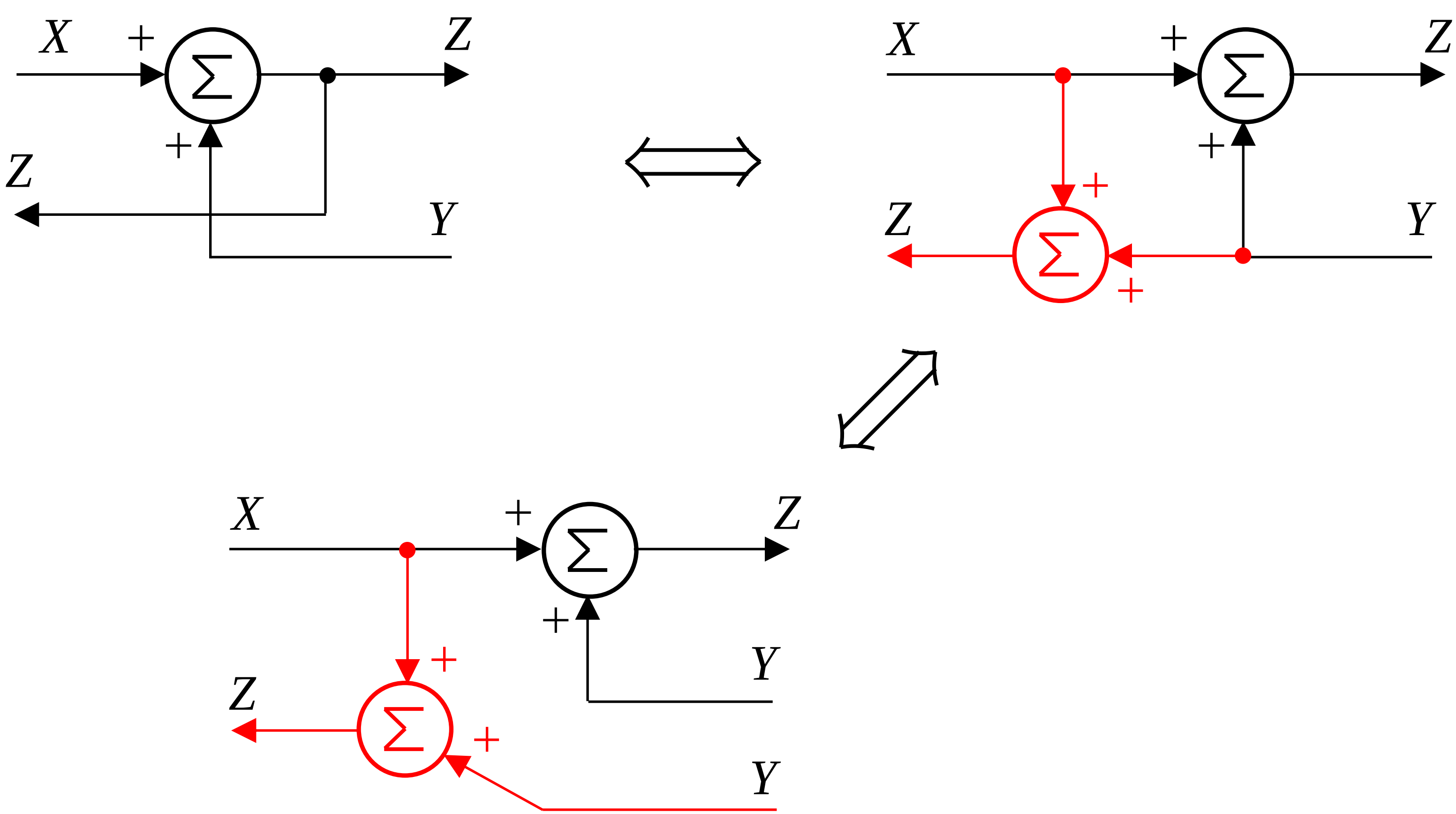
Transformation	Equation	Block diagram	Equivalent block diagram
1. Combining blocks in cascade	$Y=(G_1G_2)X$		
2. Combining blocks in parallel; or eliminating a forward path	$Y=G_1X\pm G_2X$		
3. Removing a block from a forward path	$Y=G_1X\pm G_2X$		
4. Eliminating a feedback loop	$Y=G_1(X\pm G_2Y)$		
5. Removing a block from a feedback loop	$Y=G_1(X\pm G_2Y)$		

Block Diagram Manipulation

6. Rearranging summing points	$Z = W \pm X \pm Y$ $= W \pm Y \pm X$		
7. Moving a summing point ahead of a block	$Z = GX \pm Y$		
8. Moving a summing point beyond a block	$Z = G(X \pm Y)$		
9. Moving a take-off point ahead of a block	$Y = GX$		
10. Moving a take-off point beyond a block	$Y = GX$		

Block Diagram Manipulation

Another, often overlooked, transformation rule:



Block Diagram Manipulation

- Observation: What makes block diagram complicated is not so much the number of *blocks* but rather the number of *nodes* and summers present in a given configuration.
- Objective: Move blocks according to block diagram manipulation rules in order to *reduce the number of nodes/summers* by combining adjacent nodes/summers. Occasionally nodes/summers need to be split or commuted to achieve further reduction. (Example below.)

Block Diagram Manipulation

Example 4.2 (Burns)

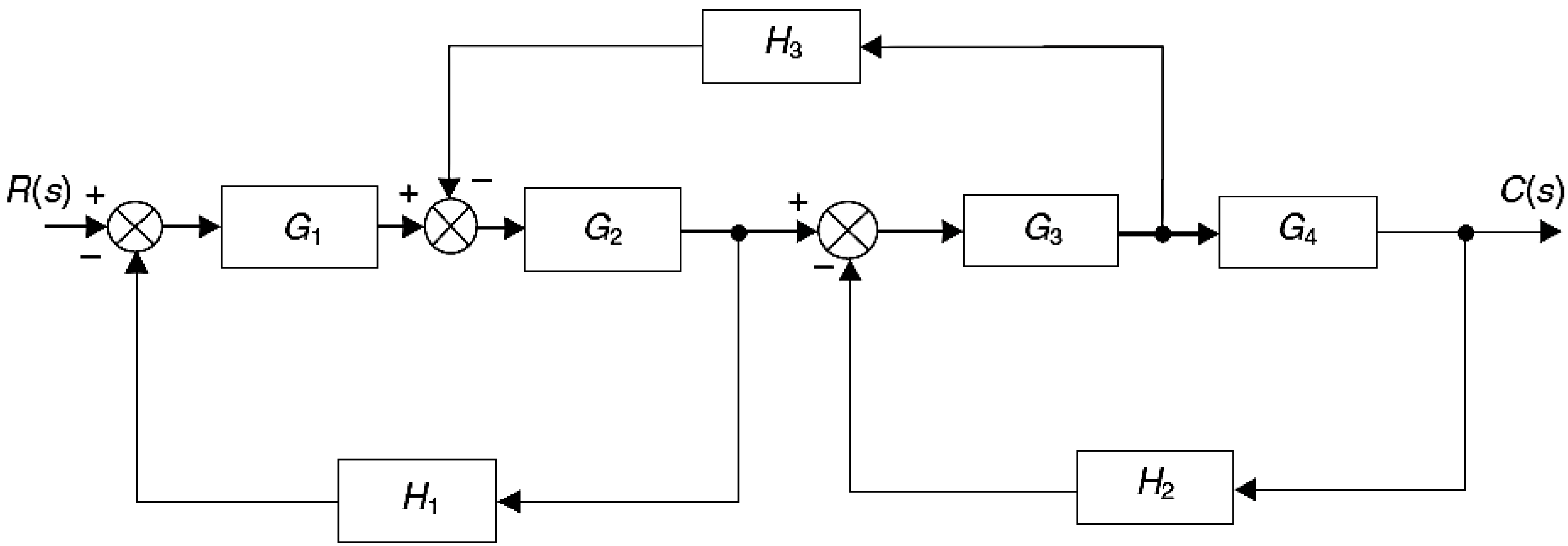


Figure 4.6 (Burns)

Block Diagram Manipulation

Example 4.2 (Burns)

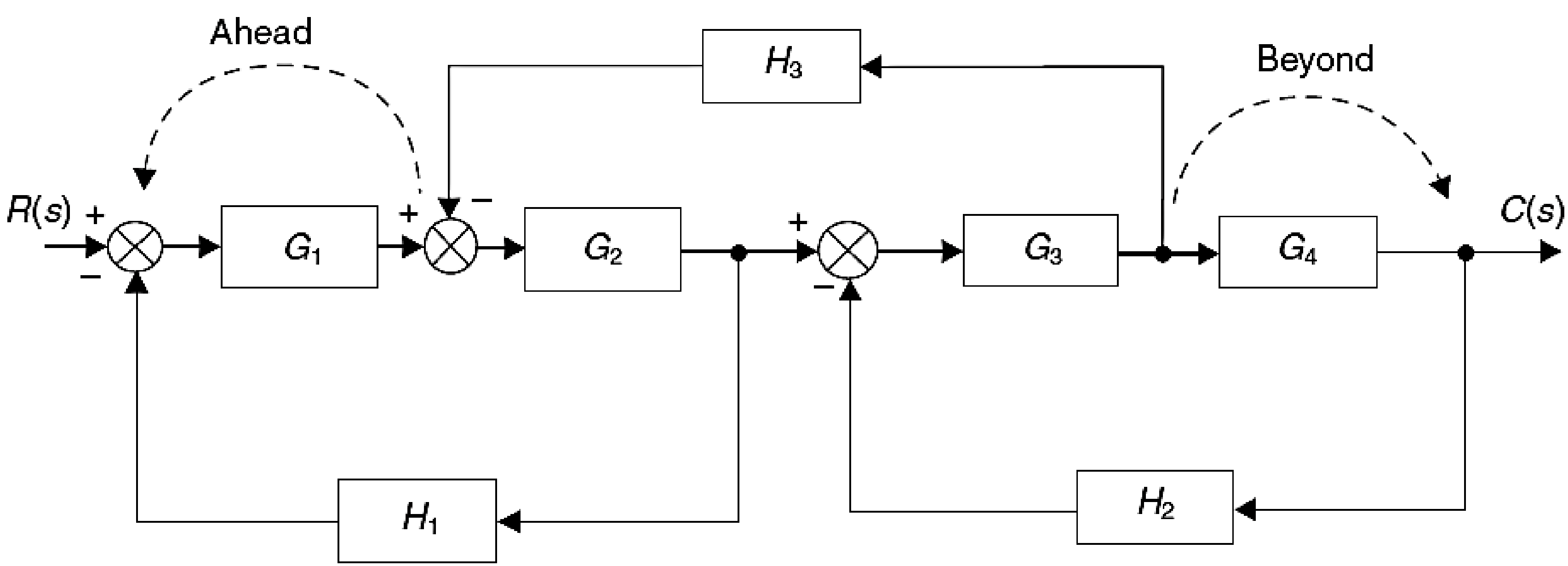


Figure 4.6 (Burns)

Block Diagram Manipulation

Example 4.2

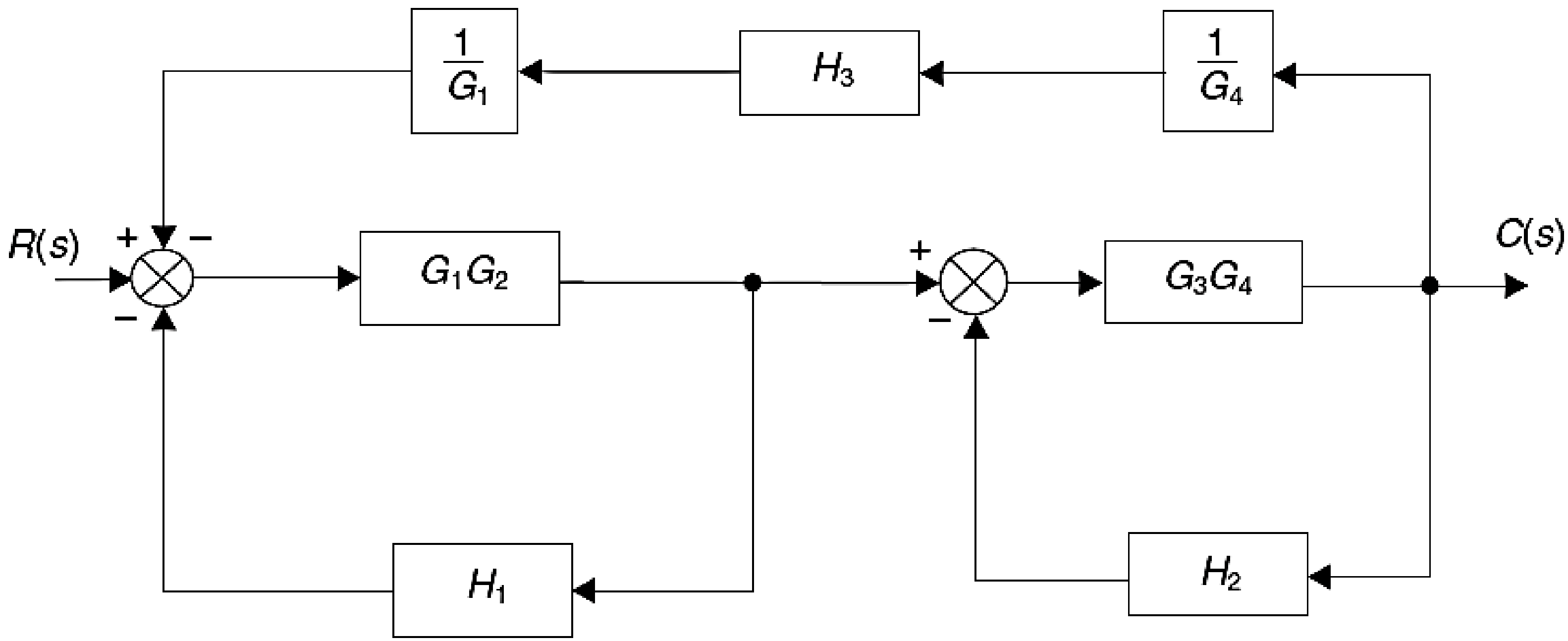


Figure 4.7 (Burns)

Block Diagram Manipulation

Example 4.2

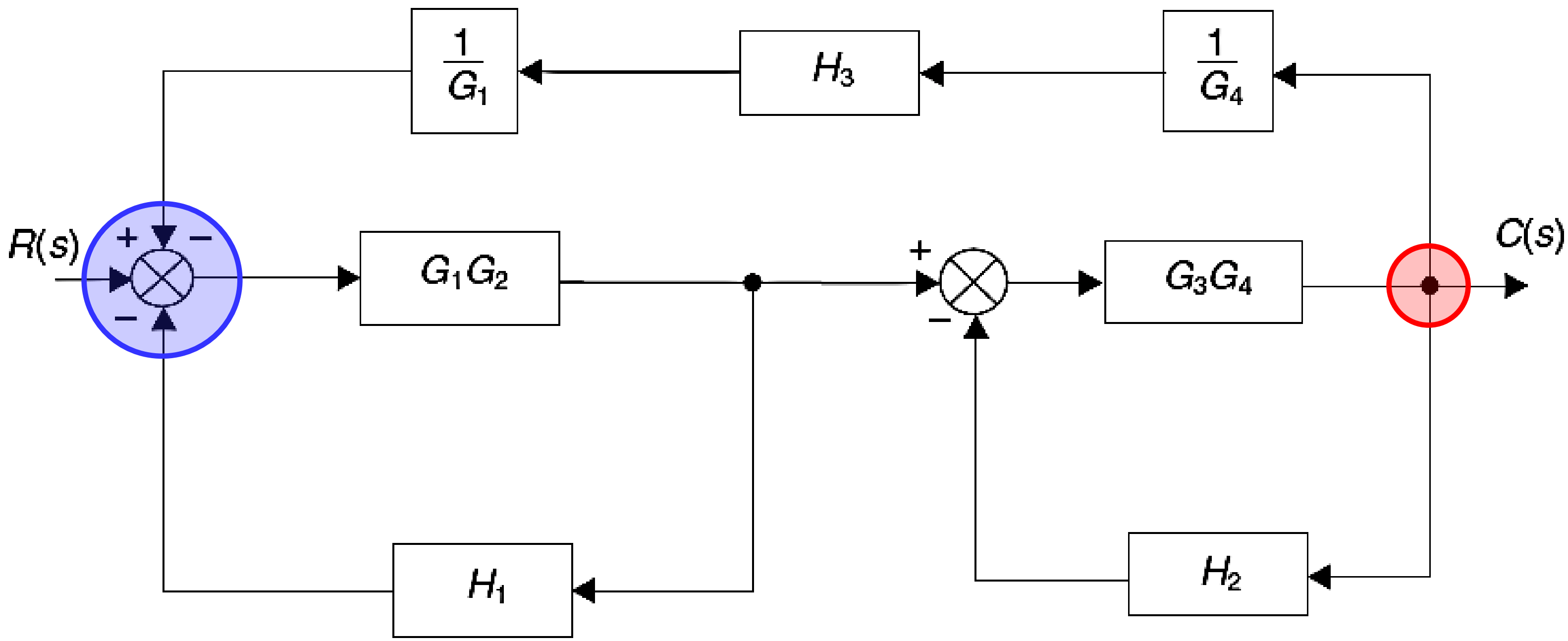


Figure 4.7 (Burns)

Block Diagram Manipulation

Example 4.2

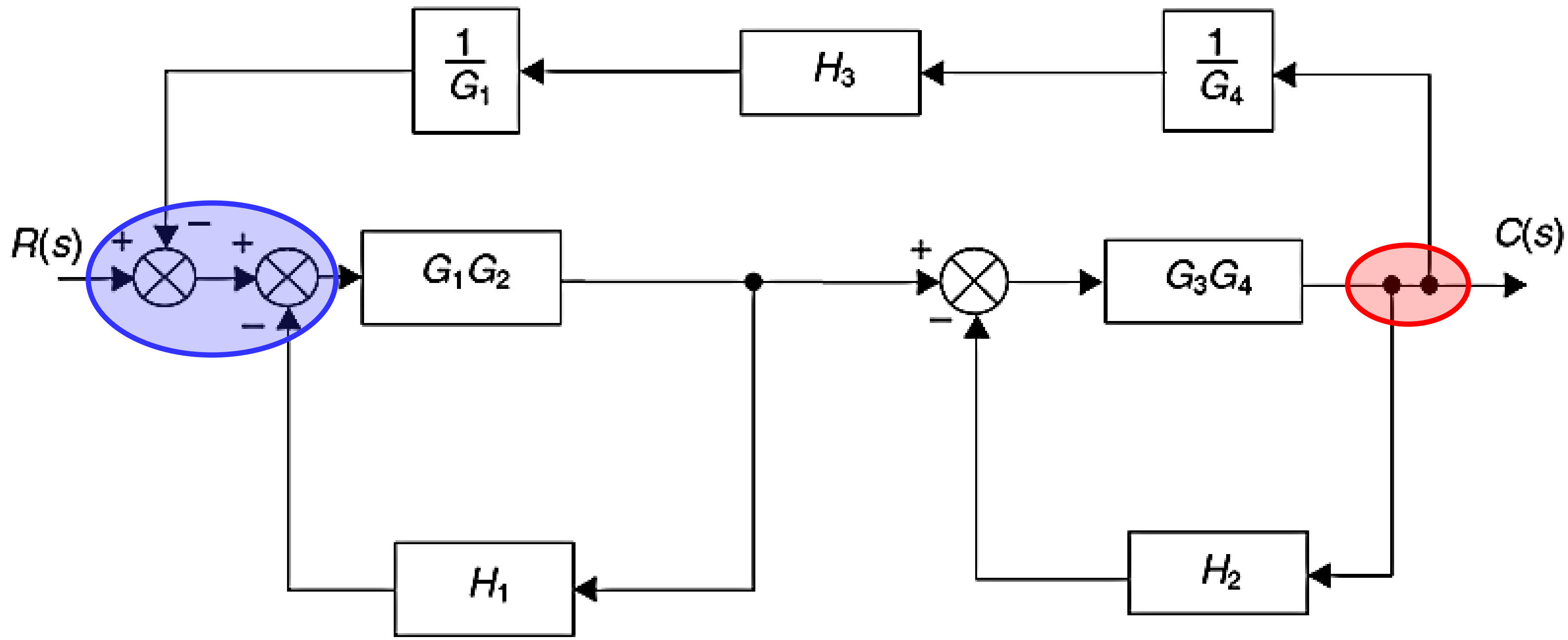


Figure 4.7 (Burns)

Block Diagram Manipulation

Example 4.2

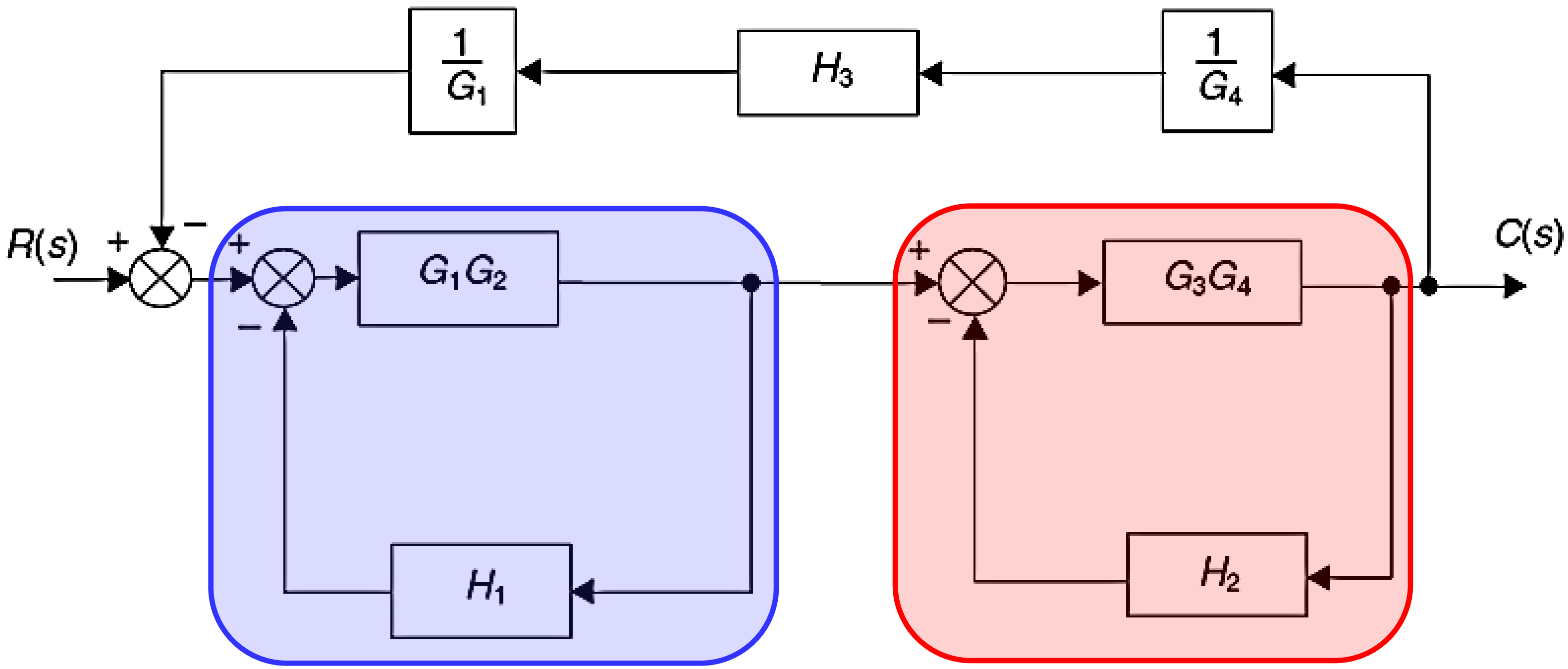


Figure 4.7 (Burns)

Block Diagram Manipulation

Example 4.2

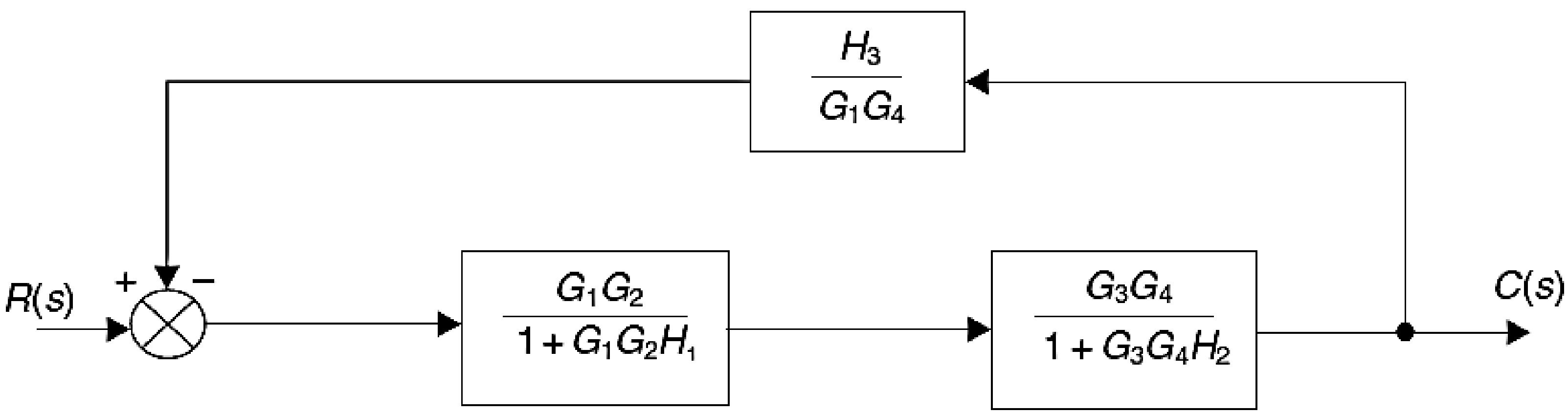


Figure 4.8 (Burns)

Tutorial Exercises & Homework

- Tutorial Exercises
  - None
- Homework
  - Examples in Burns not covered in class.

Conclusion

- Closed-Loop Systems
- Block Diagram Manipulation
- Some Examples
- Superposition (Self-study!)
- Examples not covered (Self-study!)
- Section 4.4.2 (Omit)
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- More examples using Block Diagram Manipulation/Algebra

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Thank you!

Any Questions?