CONTROL I

ELEN3016

Time Domain Analysis

(Lecture 5)

Overview

- First Things First!
- Time Domain Analysis (Overview)
- Examples
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

- Lab Notes
  - To be finalised Thursday.
- Misprints
  - Burns: p 52; Fig 3.18, p 53; p 54; Eq (3.68), p 56; p 57.

Time-Domain Analysis

- Linear System Output (from Signals & Systems)

$$x_o(t) = \underbrace{\text{Zero-Input Response}}_{\text{Due to initial conditions}} + \underbrace{\overbrace{\text{Zero-State Response}}^{\text{Transient Response + Forced Response}}}_{\text{Due to convolution of input \& impulse response}} \quad \text{(Lathi, p 53)}$$

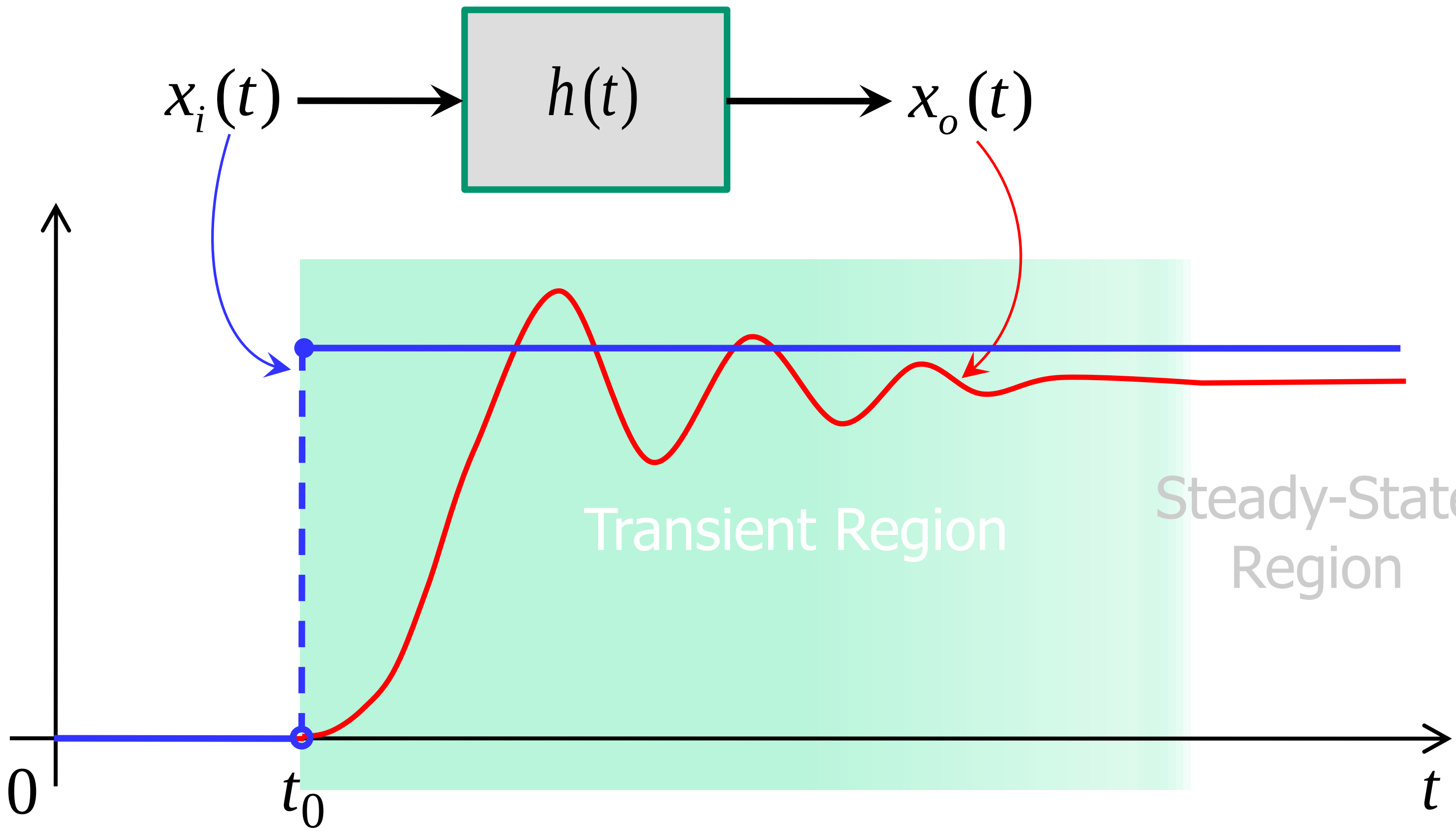
$$= \text{Natural Response} + \text{Forced Response} \quad \text{(Lathi, p 80)}$$

$$= \text{Transient Response} + \text{Steady-State Response} \quad \text{(Lathi, p 82)}$$

- For Sinusoidal Inputs
  - Transient Response = Natural Response
  - Steady-State Response = Forced Response

Time-Domain Analysis

- Transient & Steady-State Regions



Time-Domain Analysis

- Prototype 2nd-Order Transfer Function

$$a\,\ddot{x}_o(t) + b\,\dot{x}_o(t) + c\,x_o(t) = e\,x_i(t)$$

$$G(s) = \frac{X_o(s)}{X_i(s)} = \frac{e}{as^2 + bs + c}$$

$$= \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Time-Domain Analysis

- Prototype 2nd-Order Transfer Function

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?

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Time-Domain Analysis

- Prototype 2nd-Order Transfer Function

$$K = \frac{e}{c}$$ Steady-state gain

$$\omega_n = \sqrt{\frac{c}{a}}$$ Natural frequency

$$\zeta = \frac{1}{2}\frac{b}{\sqrt{ac}}$$ Damping ratio

$$\omega_n \zeta = \frac{b}{2a}$$ Damping factor

Time-Domain Analysis

- Poles of the Prototype 2nd-Order System
  - Over-damped $(b^2 > 4ac$ or $\zeta > 1)$:

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1}$$

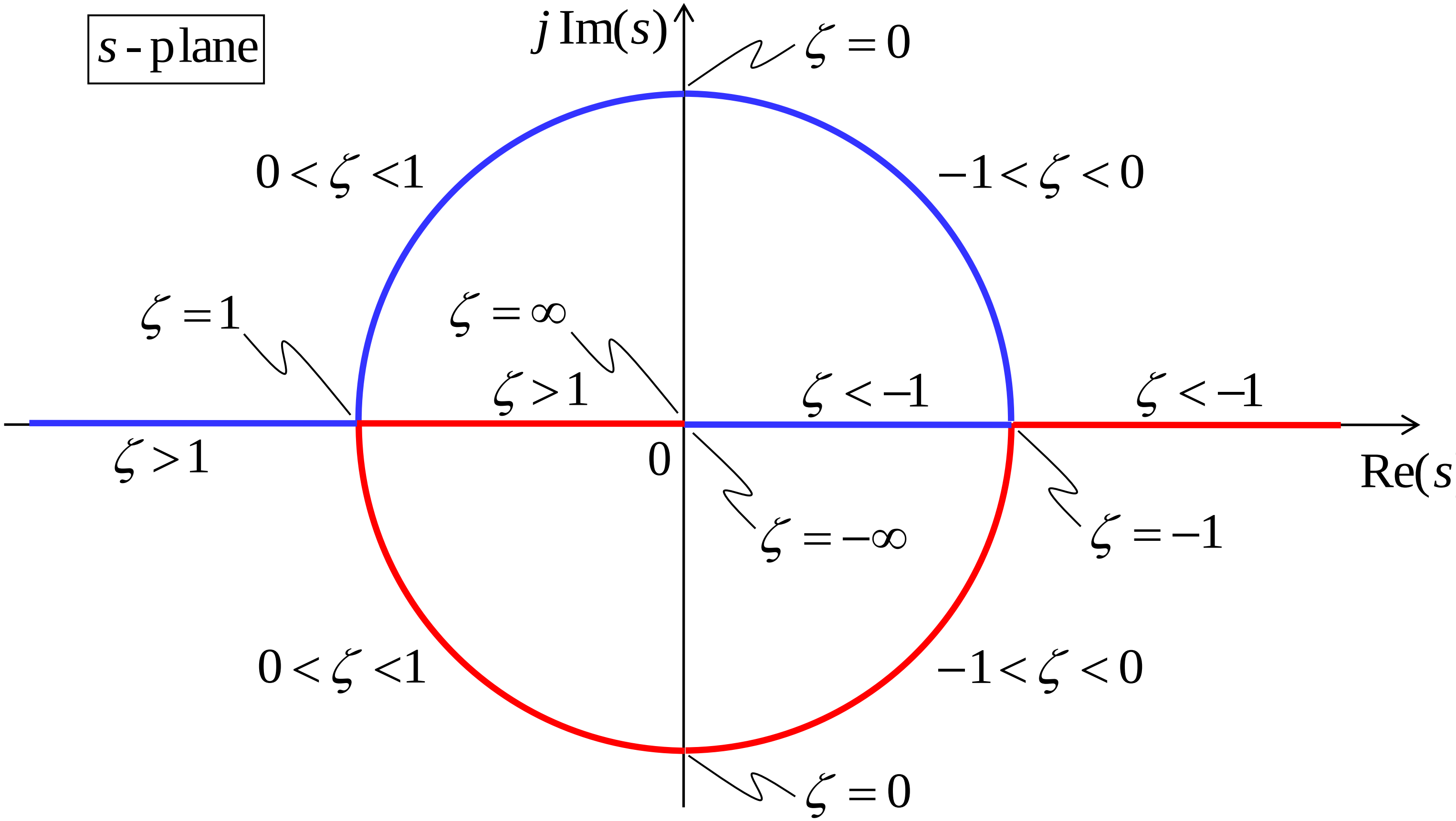
  - Under-damped $(b^2 < 4ac$ or $\zeta < 1)$:

Damped frequency, ω_d

$$s_1, s_2 = \frac{-b \pm j\sqrt{4ac - b^2}}{2a} = -\omega_n \zeta \pm j \overbrace{\omega_n \sqrt{1 - \zeta^2}}^{\omega_d}$$

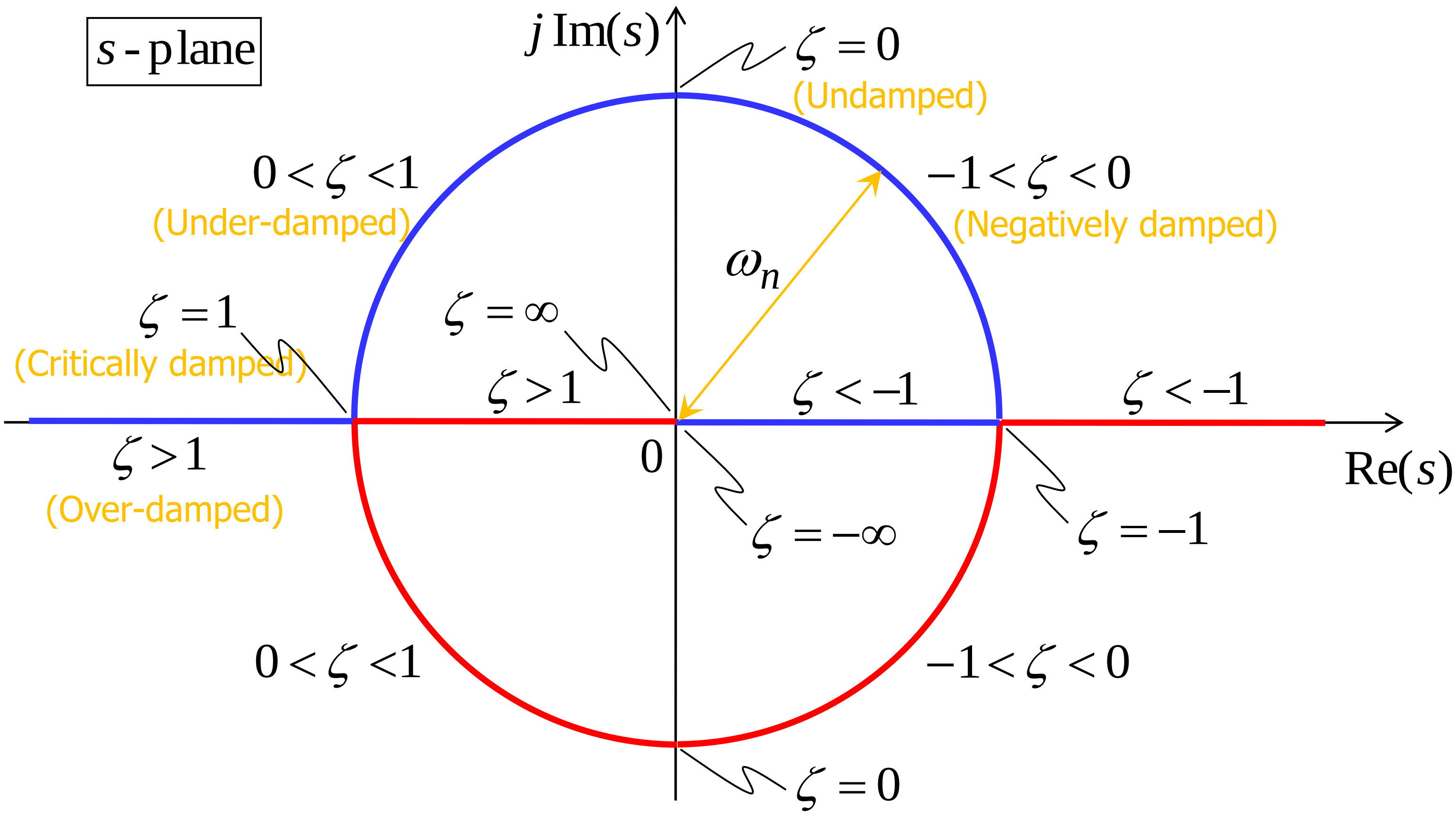
Time-Domain Analysis

- Poles of the Prototype 2nd-Order System



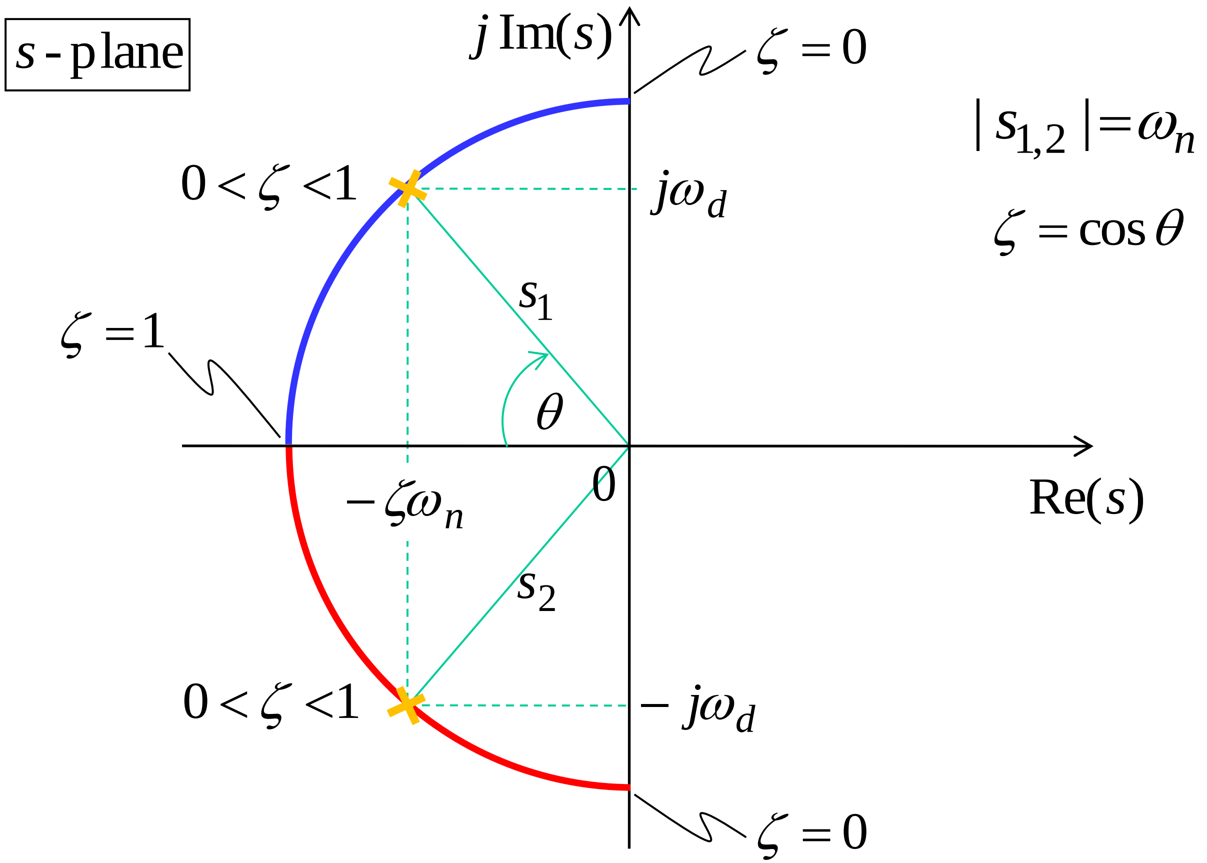
Time-Domain Analysis

- Poles of the Prototype 2nd-Order System



Time-Domain Analysis

- Poles of the Prototype 2nd-Order System



Time-Domain Analysis

- Prototype 2nd-Order System – Step Response

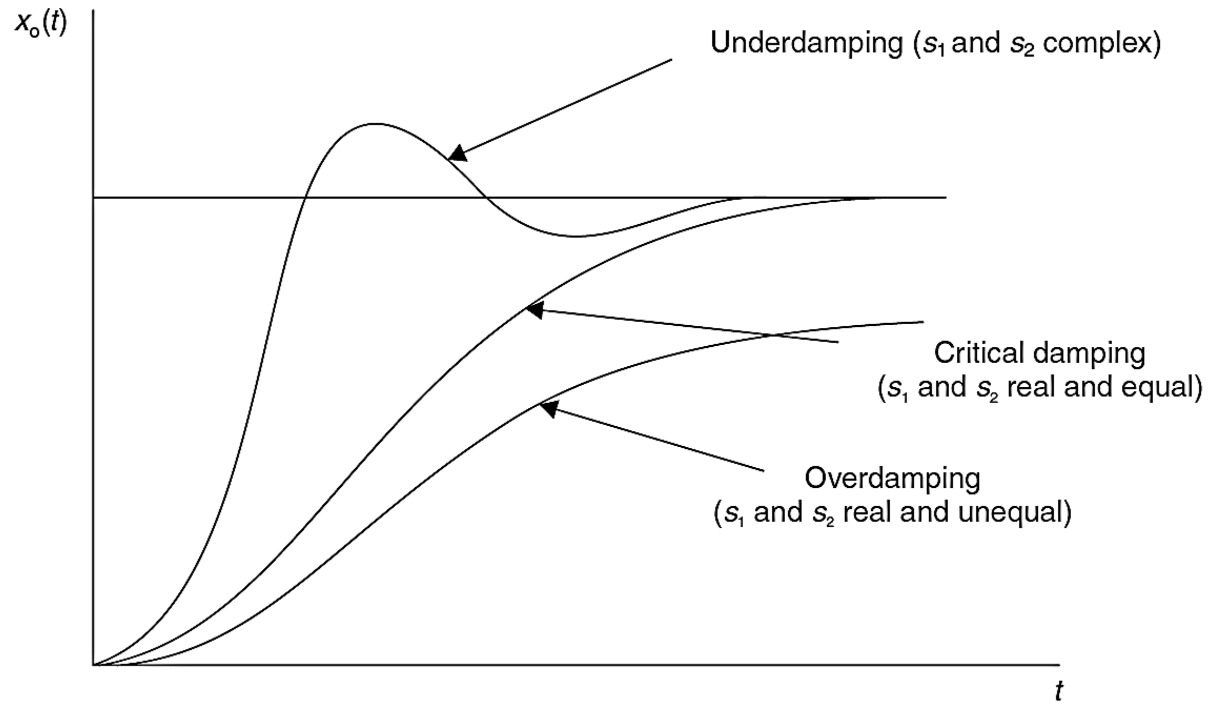


Fig. 3.16 Effect that roots of the characteristic equation have on the damping of a second-order system.

Time-Domain Analysis

- Prototype 2nd-Order System – Step Response

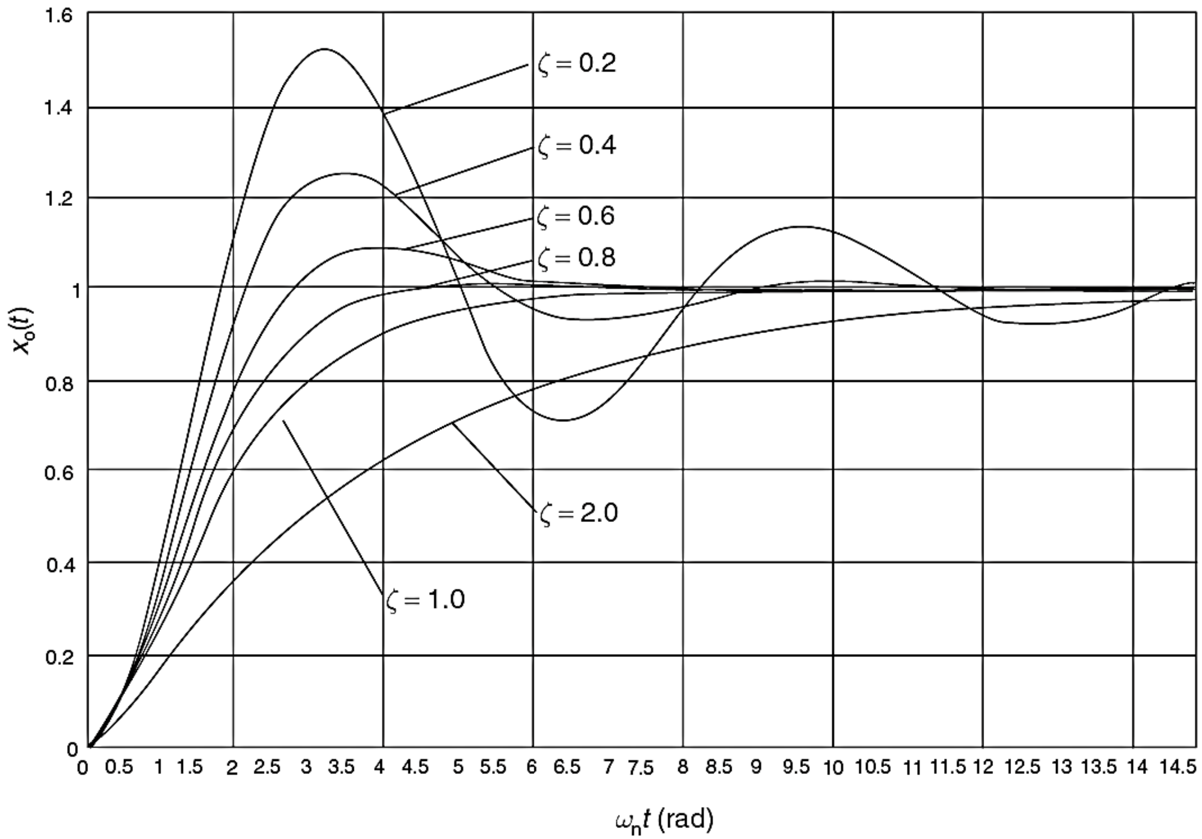


Fig. 3.19 Unit step response of a second-order system.

Time-Domain Analysis

- Prototype 2nd-Order System – Step Response
  - Refer to Burns, Figures 3.16 & 3.19.
  - Observation 1: ω_d increases (and approaches ω_n) as ζ approaches zero.
  - Observation 2: Output settles in the shortest time when $\zeta = 1$.
  - Study the derivations in Burns, Section 3.6.4.

Time-Domain Analysis

- 2nd-Order System – Step Response Analysis

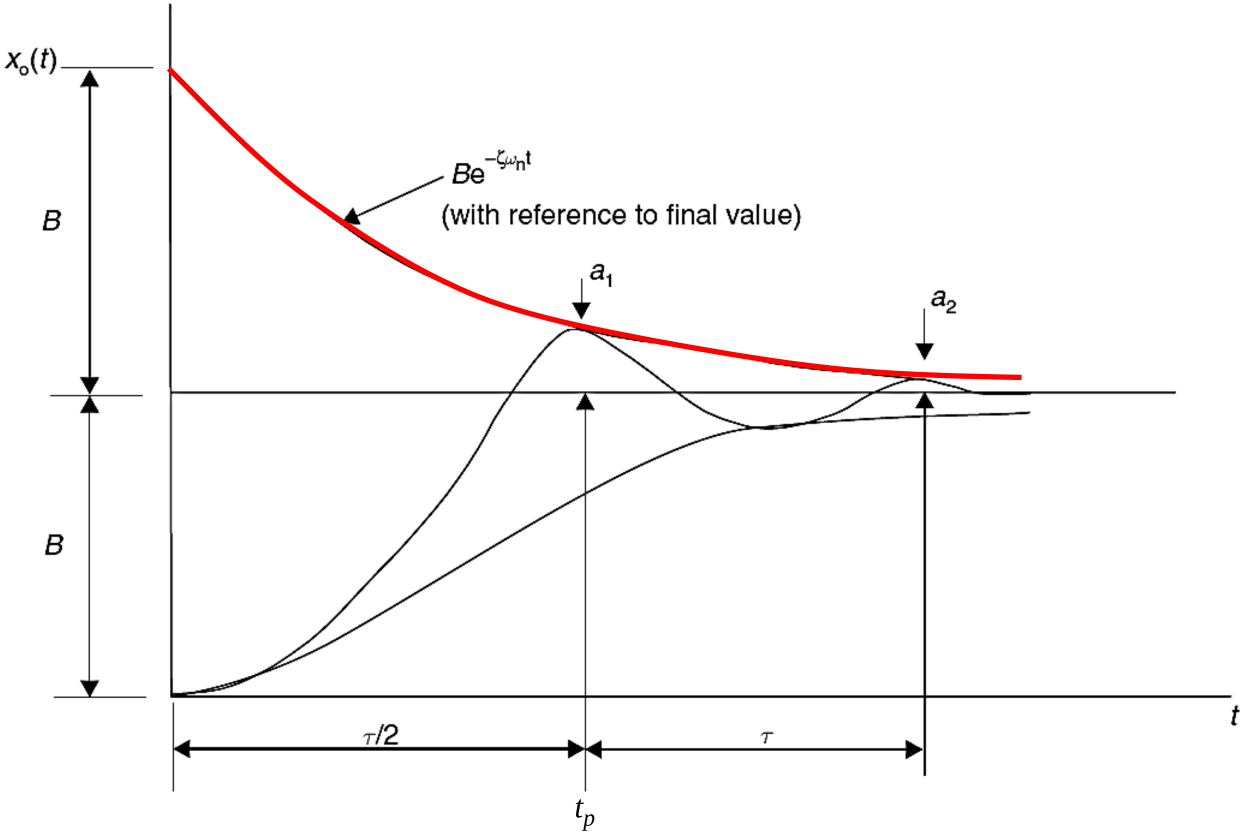


Fig. 3.20 Step response analysis.

Time-Domain Analysis

- 2nd-Order System – Step Response Analysis
  - For unity-gain $K=1$ and $x_i(t)=Bu(t)$:

$$PO=\frac{\text{Peak value - Final value}}{\text{Final value}}\times 100\% = 100\,e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$a_1 = B\,e^{-\zeta\omega_n t_p}, \qquad t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$PO=\frac{a_1}{B}\times 100, \qquad t_p = \text{Time of first peak},$$

  - Clearly ζ can be expressed i.t.o. PO.

Time-Domain Analysis

- 2nd-Order System – Step Performance Spec.
  - Rise time: $t_r = t\big|_{x_o=0.9B} - t\big|_{x_o=0.1B}$
  - 2% Settling time: $t_s = \dfrac{\ln 50}{\zeta\omega_n}$
  - Delay time: $t_d = t\big|_{x_o=0.5B}$
  - Contours in the s-plane (Kuo, *Automatic Control Systems*, 7th ed., Fig 7.15, p. 391 & Lathi, *Signals, Systems and Controls*, pp. 253–255.)

Tutorial Exercises & Homework

- Tutorial Exercises
  - Burns, Examples 3.9 and 3.10
- Web Surfing – Exploring 2nd-Order Systems

  http://www.facstaff.bucknell.edu/mastascu/econtrolhtml/SysDyn/SysDyn2.html
- Homework
  - Burns: Example 3.7, p 51, Sec 3.4, 3.5, 3.6.4, 3.8

Conclusion

- Briefly Reviewed Time Domain Analysis
- Focused on the Prototype 2nd-order system
- Burns, Sec 3.4, 3.5, 3.6.4, 3.8 (Self-study!)
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Closed-Loop Control Systems (Burns, Chapter 4)

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Thank you!

Any Questions?