CONTROL I

ELEN3016

System Modelling

(Lecture 3)

Overview

- First Things First!
- More Examples of Mechanical Systems
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

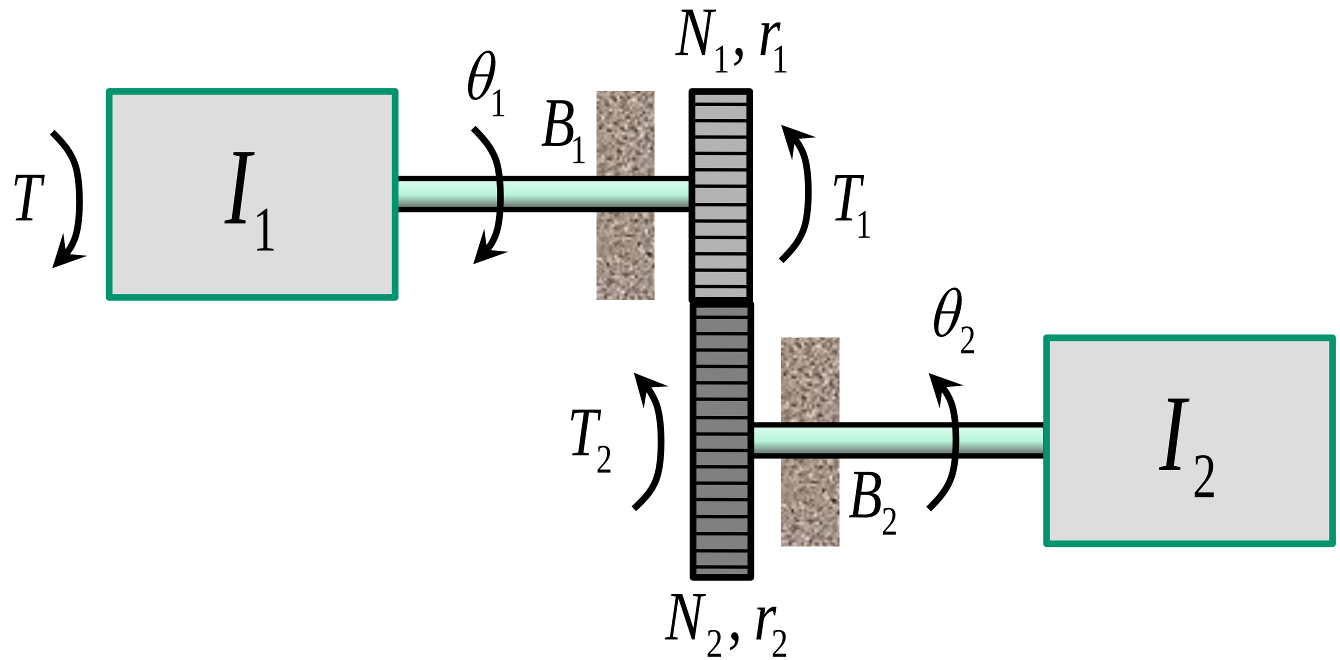
- Tut & Lecture Swap
  - Lecture on Wednesdays & Tut on Thursdays.
- Consultation
  - Wednesdays 8:30 – 10:00 AM
- Lab Matters
  - The lab brief will be available early next week.
- Lecture Notes

  http://dept.ee.wits.ac.za/~vanwyk/ELEN3016_2015/

Example – Real Gearbox

(Similar to Burns, Example 2.3)

– Input: Torque, $T(t)$ Output: Angle, $\theta_2(t)$



– Internal reaction torques: $T_1(t) = r_1 X(t)$, $T_2(t) = r_2 X(t)$

– Internal reaction force: $X(t)$ (See to Burns, Example 2.3)

Example – Real Gearbox

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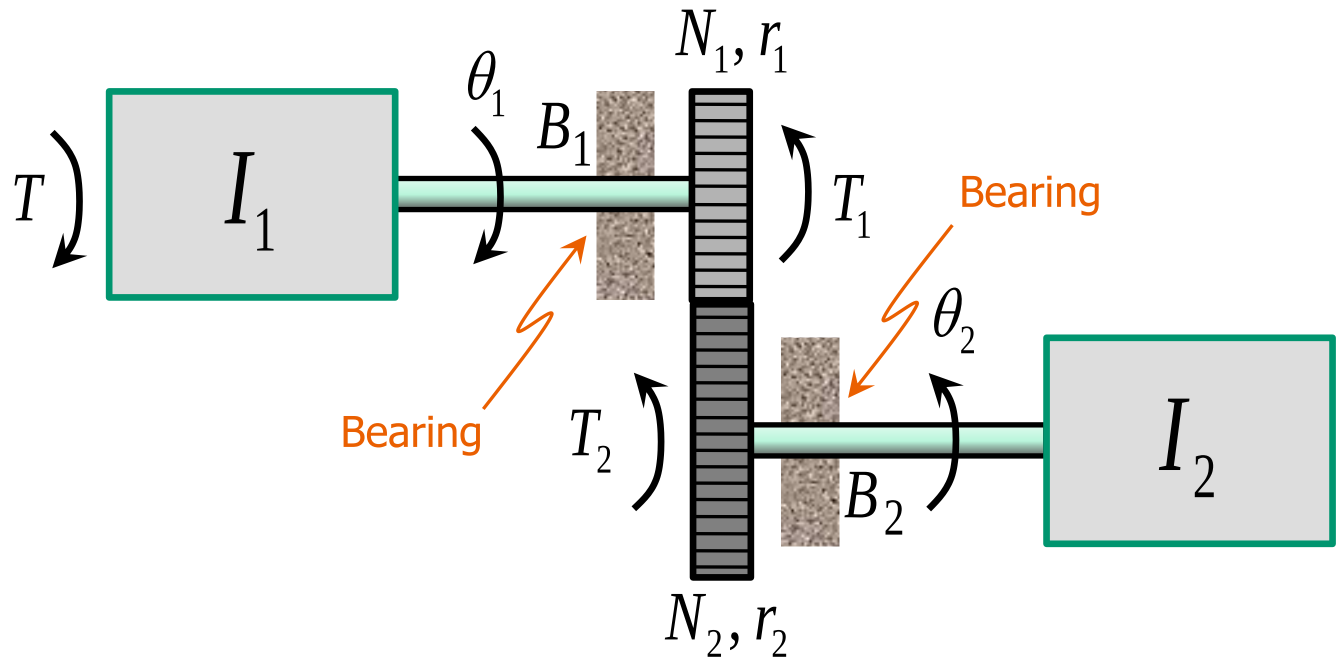

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- Internal reaction force: $X(t)$ (See to Burns, Example 2.3)

Example – Real Gearbox

- Taper Roller Bearing

For bearing terminology visit: http://www.rbcbearings.com/tapered/components.htm

Example – Real Gearbox

- Ball Bearing

Example – Real Gearbox

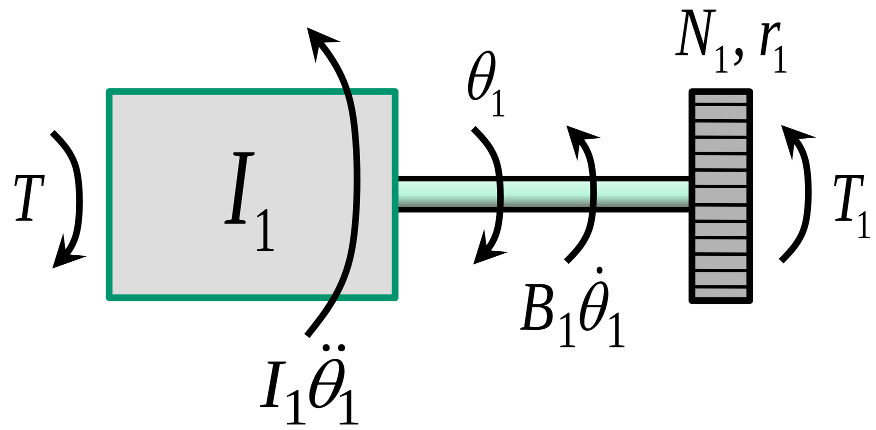
(Similar to Burns, Example 2.3)

- Simplifying assumptions:
  - We assume the gearbox to be *ideal* :
    - The inertia of each gear-shaft assembly is small compared to the inertias I_1 and I_2
    - No friction present
    - No backlash present
- Model equations:

$$\frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1}$$

Example – Real Gearbox
(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_1:



Example – Real Gearbox

(Similar to Burns, Example 2.3)

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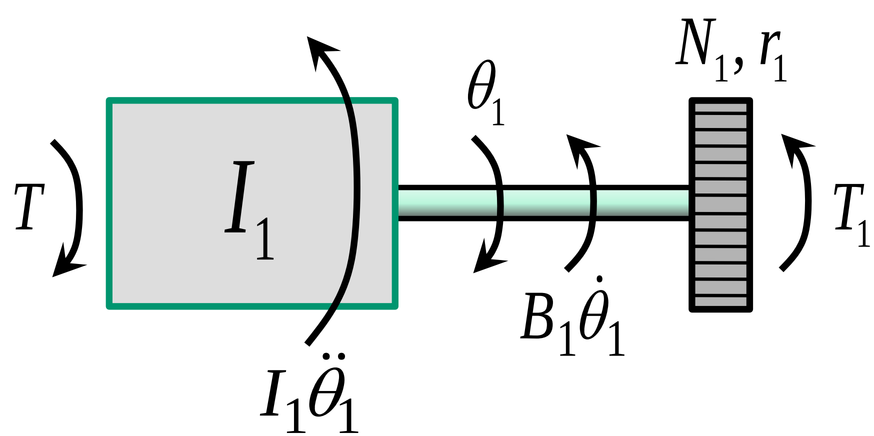
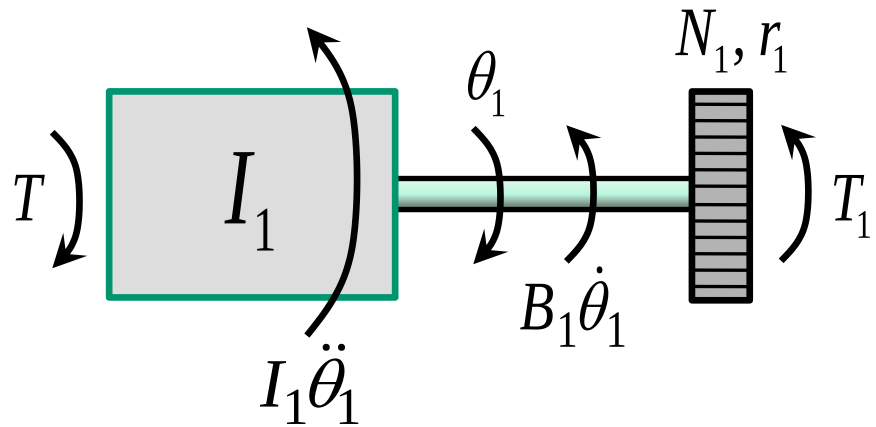


Newton's 2nd law: $T(t) - I_1\ddot{\theta}_1(t) - B_1\dot{\theta}_1(t) - T_1(t) = 0$

Example – Real Gearbox

(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_1:



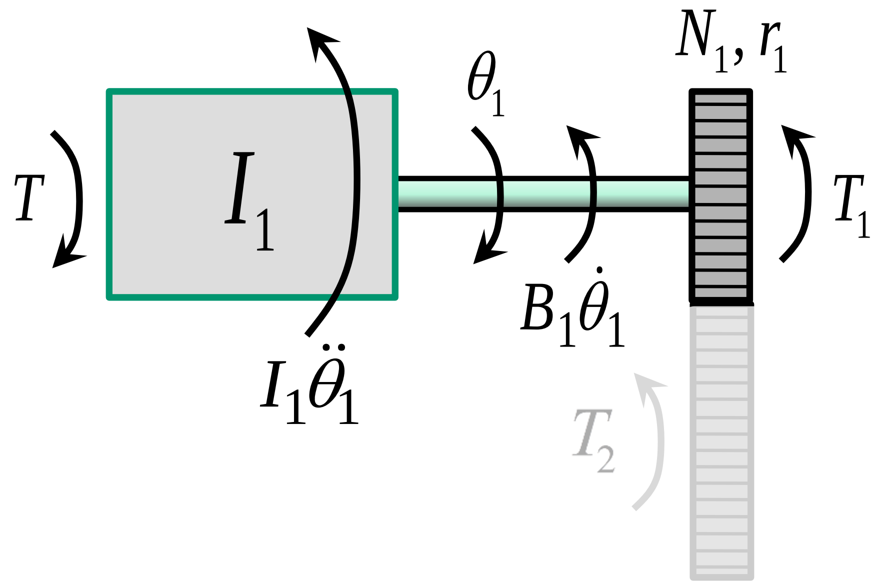
Newton's 2nd law: $T(t) - I_1\ddot{\theta}_1(t) - B_1\dot{\theta}_1(t) - T_1(t) = 0$

$$\Rightarrow \quad T(t) - I_1\ddot{\theta}_1(t) - B_1\dot{\theta}_1(t) = T_1(t)$$

Example – Real Gearbox

(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_1:



$$\frac{T_1(t)}{T_2(t)} = \frac{r_1 X(t)}{r_2 X(t)} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$\Rightarrow \quad T_1(t) = \frac{N_1}{N_2} T_2(t)$$

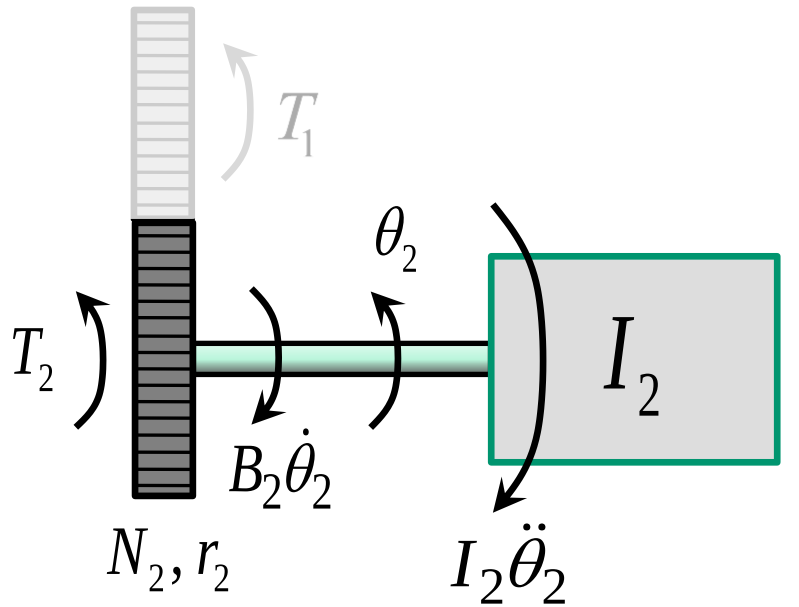
Newton's 2nd law: $T(t) - I_1\ddot{\theta}_1(t) - B_1\dot{\theta}_1(t) - T_1(t) = 0$

$$\Rightarrow \quad T(t) - I_1\ddot{\theta}_1(t) - B_1\dot{\theta}_1(t) = T_1(t)$$

Example – Real Gearbox

(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_2:

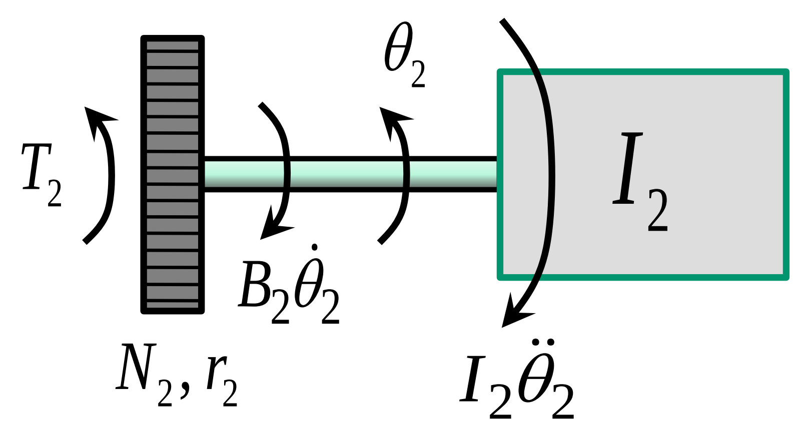


Example – Real Gearbox

(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_2:

Newton's 2nd law: $T_2(t) - I_2\ddot{\theta}_2(t) - B_2\dot{\theta}_2(t) = 0$

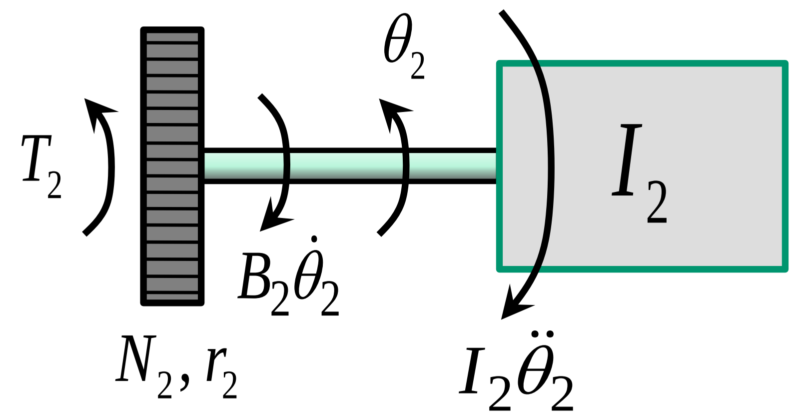


Example – Real Gearbox
(Similar to Burns, Example 2.3)

– Free-body diagrams for inertia I_2:

Newton's 2nd law: $T_2(t) - I_2\ddot{\theta}_2(t) - B_2\dot{\theta}_2(t) = 0$

$$\Rightarrow \quad I_2\ddot{\theta}_2(t) + B_2\dot{\theta}_2(t) = T_2(t)$$



Example – Real Gearbox

(Similar to Burns, Example 2.3)

- Recalling that $\theta_2(t) = \frac{N_1}{N_2}\theta_1(t)$ and combining with the above two equations yields

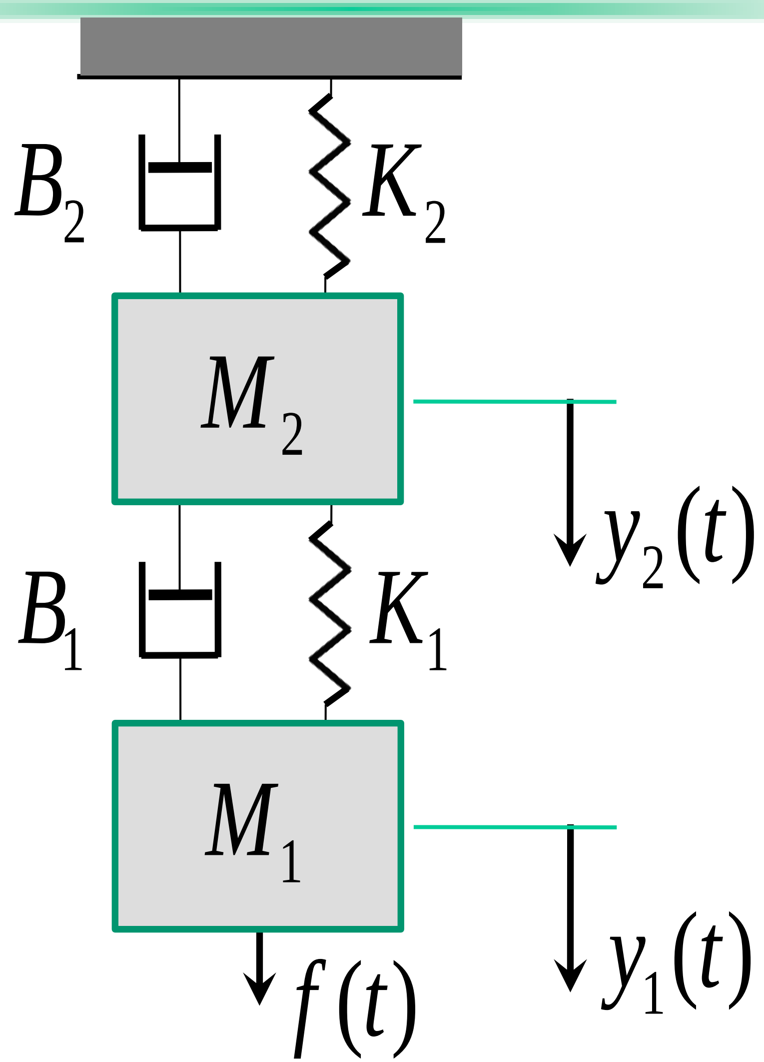
$$\left(I_1 + \left(\frac{N_1}{N_2}\right)^2 I_2 \right)\ddot{\theta}_1(t) + \left(B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 \right)\dot{\theta}_1(t) = T(t)$$

or simply

$$\tilde{I}_1\ddot{\theta}_1(t) + \tilde{B}_1\dot{\theta}_1(t) = T(t).$$

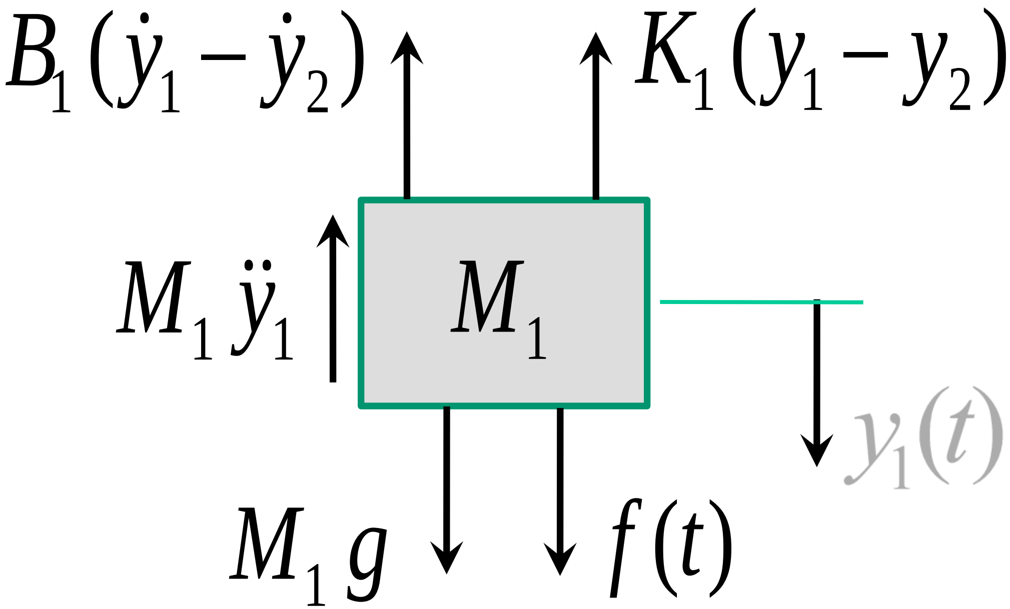
- Parameters: $N_1, N_2, I_1, I_2, B_1, B_2$

Example – Mechanical System



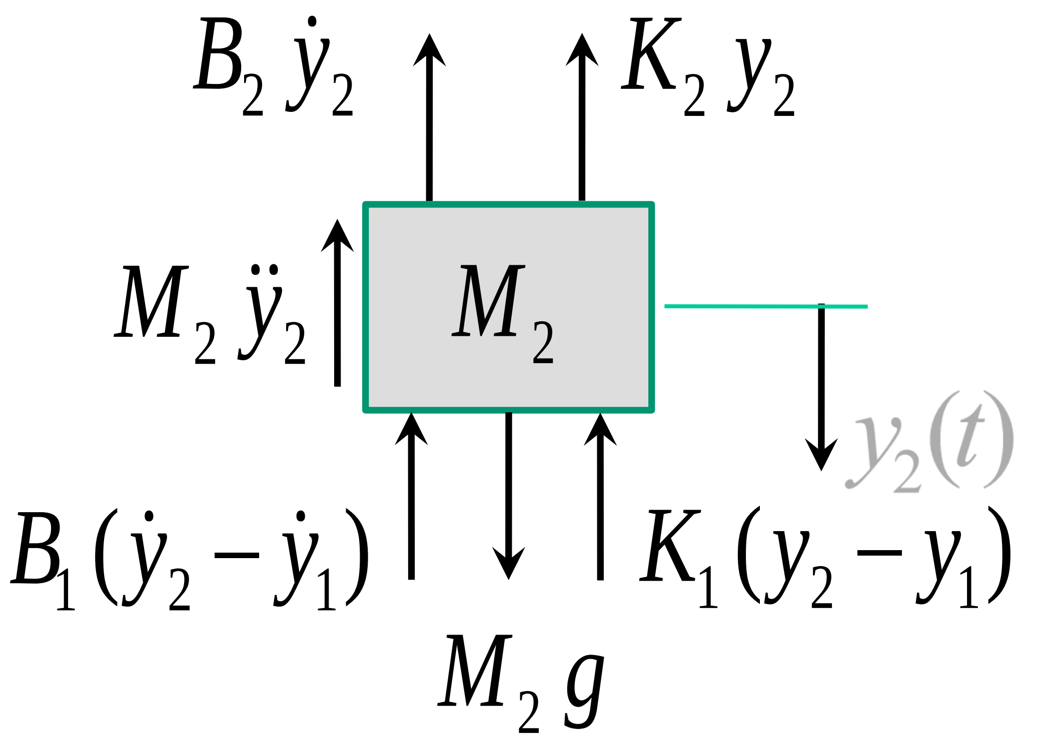
Example – Mechanical System

Free-body diagram for Mass 1:



Example – Mechanical System

Free-body diagram for Mass 2:



Example – Mechanical System

- Analysis – NII for Mass 1:

$$-M_1\ddot{y}_1 - B_1(\dot{y}_1 - \dot{y}_2) - K_1(y_1 - y_2) + M_1 g + f(t) = 0$$

$$M_1\ddot{y}_1 + B_1\dot{y}_1 + K_1 y_1 - B_1\dot{y}_2 - K_1 y_2 = M_1 g + f(t)$$

- Analysis – NII for Mass 2:

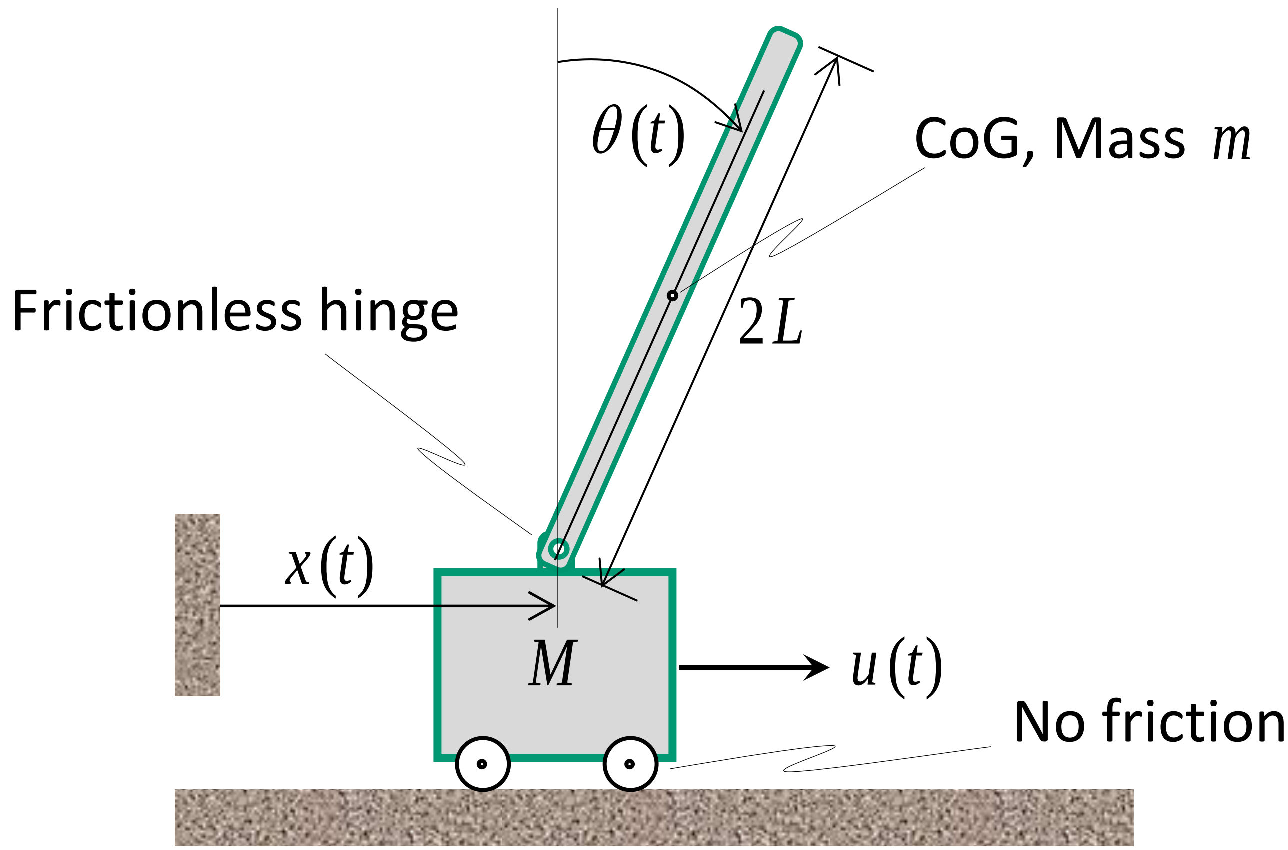
$$-M_2\ddot{y}_2 - B_2\dot{y}_2 - K_2 y_2 - B_1(\dot{y}_2 - \dot{y}_1)$$
$$- K_1(y_2 - y_1) + M_2 g = 0$$

$$-B_1\dot{y}_1 - K_1 y_1 + M_2\ddot{y}_2 + (B_1 + B_2)\dot{y}_2 + (K_1 + K_2)y_2 = M_2 g$$

Example – Inverted Pendulum

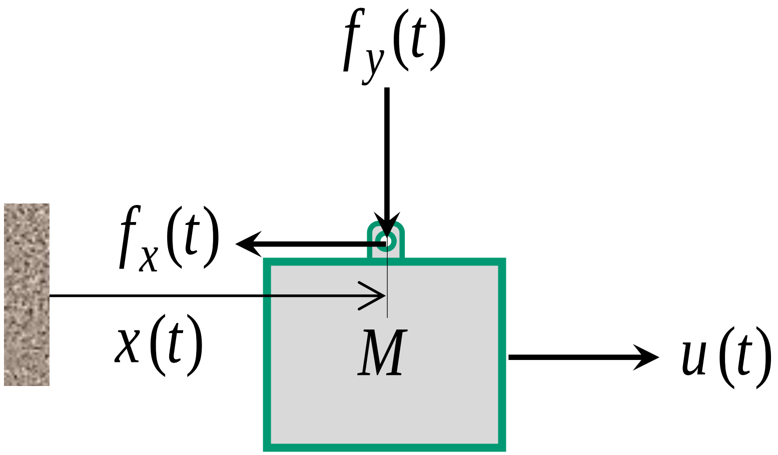
(Similar to Burns, Example 10.3)

– Input: Force, $u(t)$ Output: Angle, $\theta(t)$



Example – Inverted Pendulum

Free-body diagram for the car:

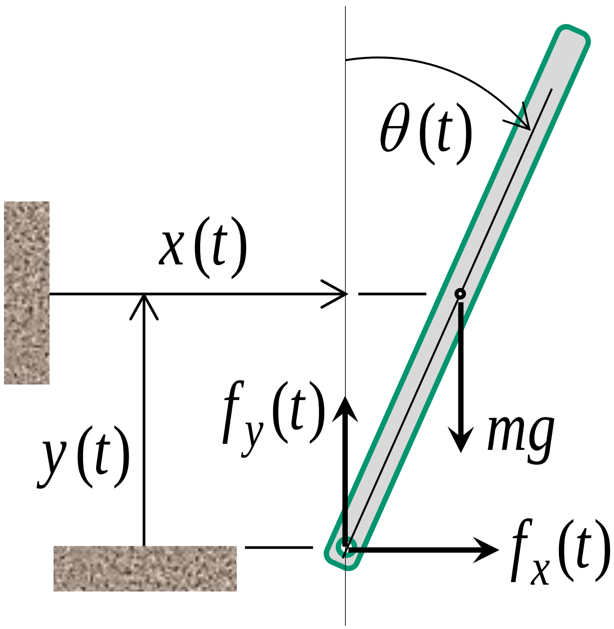


Newton's 2nd law, x-direction:

$$M\ddot{x} = u - f_x$$

Example – Inverted Pendulum

Free-body diagram for the pendulum:



$$I = \tfrac{1}{3} m L^2$$

NII of CoG in *x*-direction:

$$m \frac{d^2}{dt^2}\left(x + L \sin \theta\right) = f_x$$

NII of CoG in *y*-direction:

$$m \frac{d^2}{dt^2}\left(L \cos \theta\right) = f_y - mg$$

NII for rotation about CoG:

$$I \frac{d^2\theta}{dt^2} = f_y L \sin \theta - f_x L \cos \theta$$

Example – Inverted Pendulum

- Eliminating the reaction forces amongst these three equations yield two equations:

$$(m+M)\ddot{x}+\tfrac{1}{2}mL\cos\theta\,\ddot{\theta}-\tfrac{1}{2}mL\sin\theta\,\dot{\theta}^2=u(t)$$

$$\tfrac{1}{2}mL\cos\theta\,\ddot{x}+\tfrac{1}{3}mL^2\,\ddot{\theta}-\tfrac{1}{2}mgL\sin\theta=0$$

Example – Inverted Pendulum

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$$\tfrac{1}{2}mL\cos\theta\,\ddot{x}+\tfrac{1}{3}mL^2\,\ddot{\theta}-\tfrac{1}{2}mgL\sin\theta=0$$

... perform this step now.

Tutorial Exercises & Homework

- Tutorial Exercises
  - Inverted pendulum with linear friction B between cart and floor and torsional friction b in the hinge.
  - A mechanical assembly (below) consists of two masses: a block of mass M_1 and a ball with mass M_2, radius R_2 and moment of inertia I_2 about the centre of mass, rolling freely along the block M_1 without slipping. The block is connected to the wall by a spring K. A force $f(t)$ is applied to the block. The frictional damping constant between the table and the block is B.

    Derive the equations describing the dynamics of this system.

Tutorial Exercises & Homework

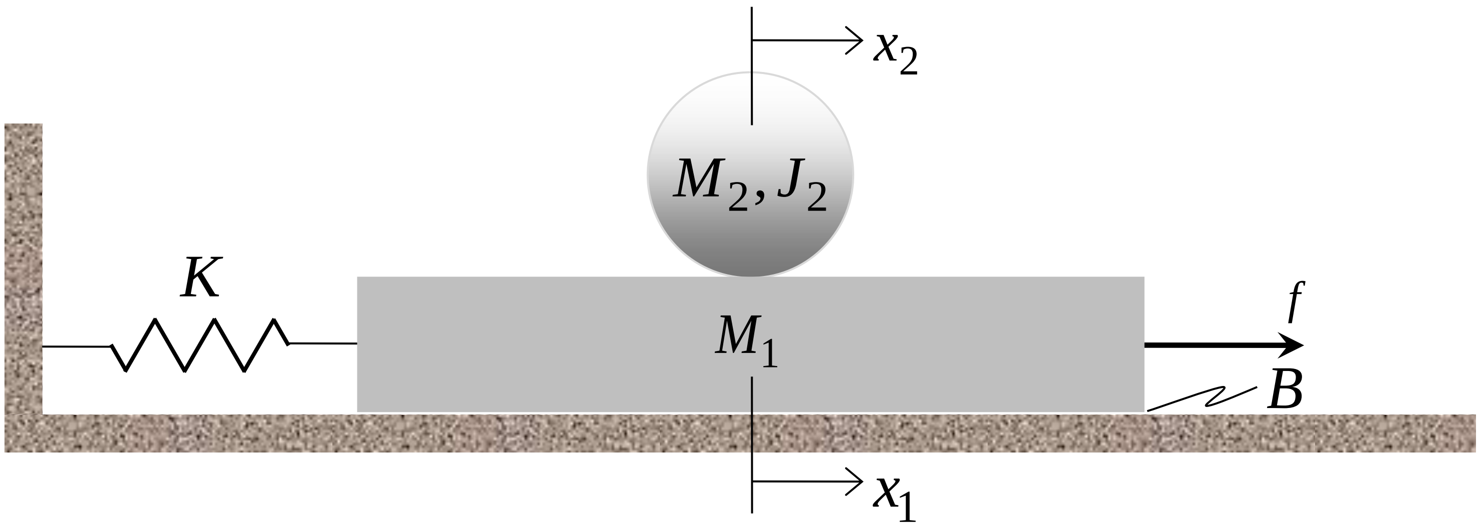

Figure: Mechanical assembly.

- Homework
  - None

Conclusion

- Example – Rotational Mechanical System
- Example – Translational Mechanical System
- Tutorial Exercises & Homework
- Notation/Conventions – vary from Book to Book

Next Attraction! – Miss It & You'll Miss Out!

- Electromechanical Energy Conversion
- Example – Brushed PM DC Motor

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Thank you!

Any Questions?