CONTROL I

ELEN3016

System Modelling

(Lecture 2)

Overview

- First Things First!
- What is Mathematical Modelling?
- Electrical Devices & Systems
- Mechanical Devices & Systems
- Examples
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

- Tut & Lecture Swap
  - Lecture on Wednesdays & Tut on Thursdays
- Consultation
  - Wednesdays 8:00 – 11:00 AM
- MATLAB Commands
  - Lists of commands are available on the internet.
- Course Homepage
  - http://dept.ee.wits.ac.za/~vanwyk/ELEN3016_2016

First Things First!

- Laboratory
  - Labs will focus on system modelling, control systems design & simulation, lab experimentation.
  - Due dates to be decided.
  - Suggested simulation software:

    Modelling & Design: MATLAB, SCILAB & OCTAVE

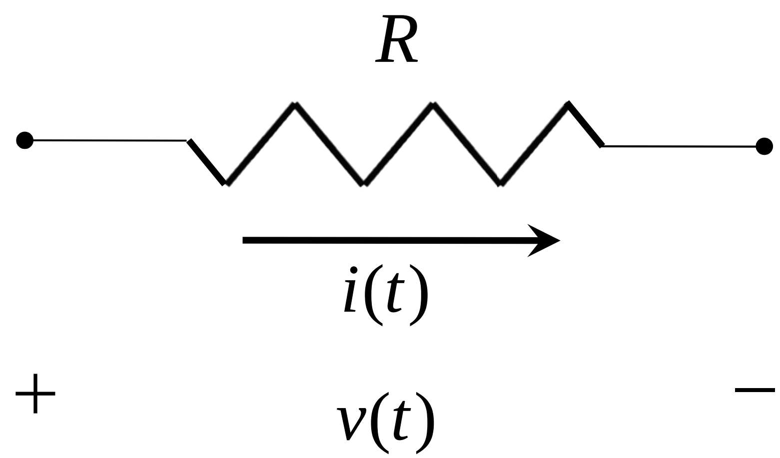
    Electrical circuits: PSPICE & MULTISIM

Mathematical Modelling

- System modelling is a "bottom-up" process
  - Quantitatively model primitive components using parametric models.
  - Combine primitive component models using the *cause-effect* principle.
  - Simplify the overall system model by imposing *simplifying arguments* (*linearity*, *frictionless*, *massless*, *differentiability*, ...)
- Knowledge of physical characteristics → parametric models

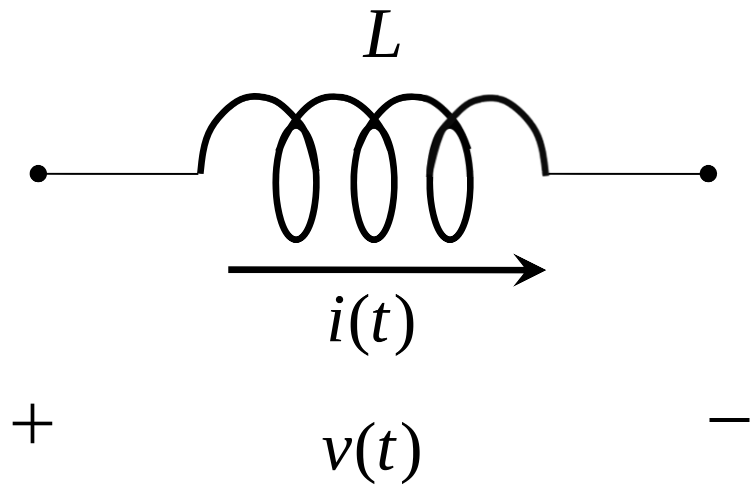
Electrical Devices

- Resistor
  - Physical characteristics: $v(t) = i(t)\,R$ (Ohm's law)
  - Parameter: R (Resistance, Vs/C)



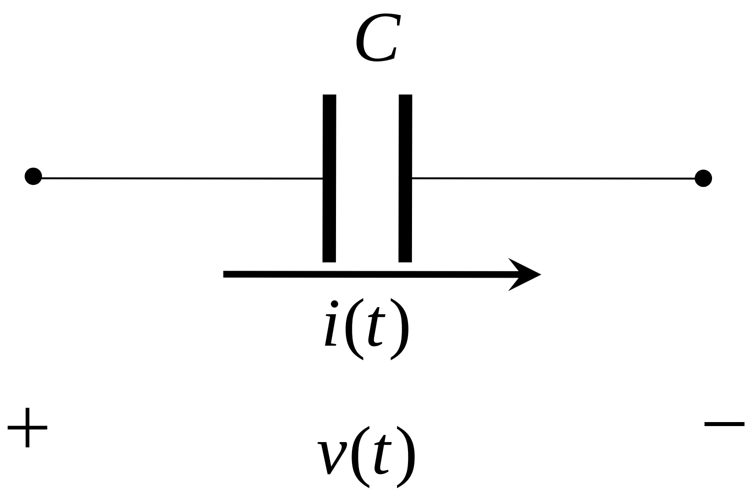
Electrical Devices

- Inductor
  - Physical characteristics: $v(t) = L\dfrac{d\,i(t)}{dt}$
  - Parameter: L (Inductance, Vs^2/C)



Electrical Devices

- Capacitor
  - Physical characteristics: $i(t) = C\dfrac{d\,v(t)}{dt}$
  - Parameter: C (Capacitance, C/V)



Electrical Systems

- Kirchhoff's Current Law

$$\sum_k i_k(t) = 0$$

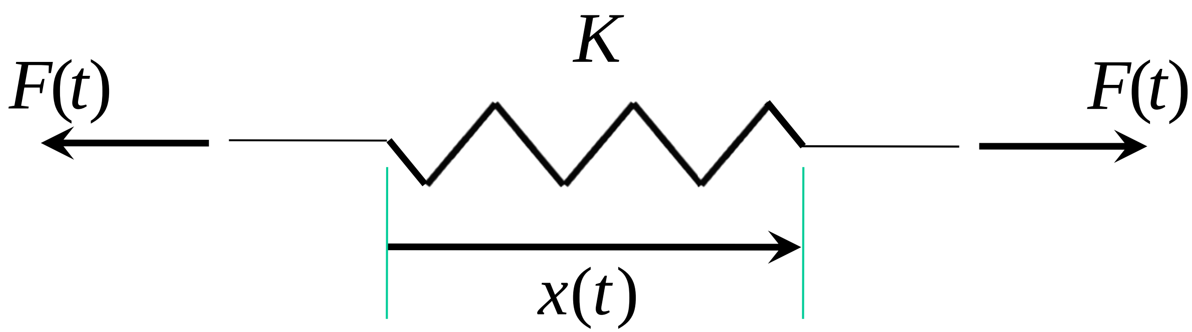
- Kirchhoff's Voltage Law

$$\sum_k v_k(t) = 0$$

- Controlled voltage & current sources
- Variable resistors, inductors & capacitors

Mechanical Devices

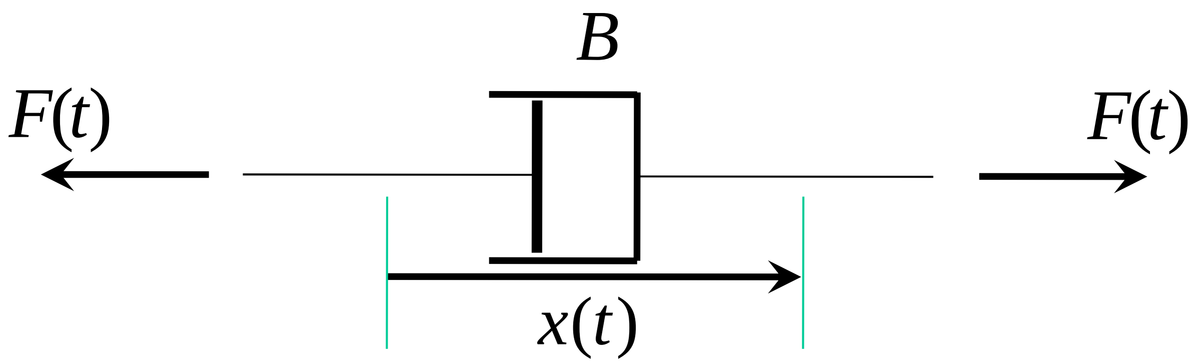
- Spring
  - Physical characteristics: $F(t) = K\,x(t)$ (Hooke's law)
  - Parameter: K (Stiffness, N/m)



  - Spot the peculiarity of this lumped parameter model!

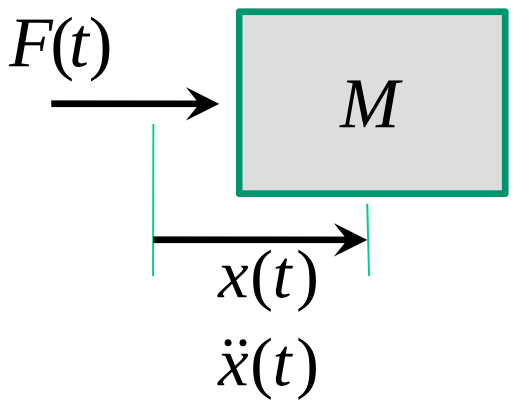
Mechanical Devices

- Damper
  - Physical characteristics: $F(t) = B\dfrac{d\,x(t)}{dt}$
  - Parameter: B (Damping coefficient, Ns/m)



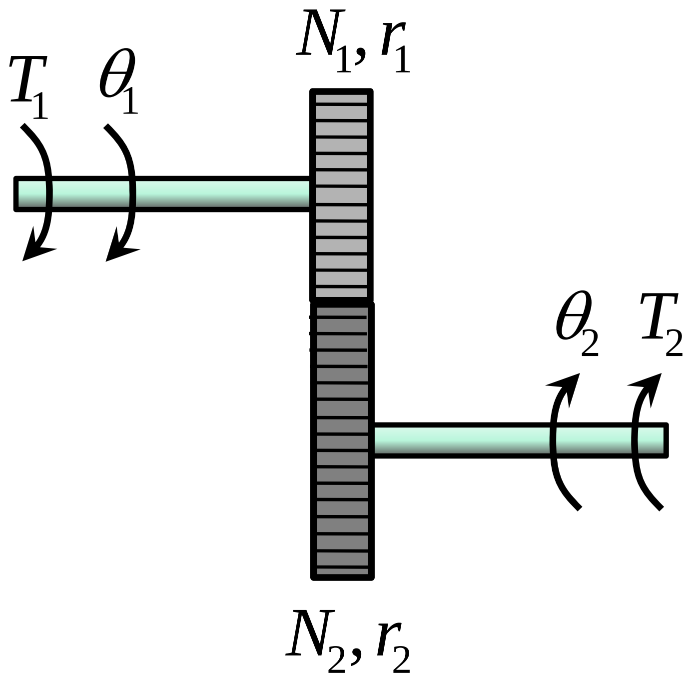
Mechanical Devices

- Mass
  - Physical char.: $F(t) = M\dfrac{d^2x(t)}{dt^2}$ (Newton's 2nd law)
  - Parameter: M (Mass, kg)



Mechanical Devices

- Two-Port (Lossless) Gearbox



  – Here T_1 and T_2 are both net applied torques.

Mechanical Devices

- Two-Port (Lossless) Gearbox (cont'd)
  - Work conservation: $T_1\theta_1 = T_2\theta_2$
  - No. of teeth to radius: $\dfrac{N_1}{r_1} = \dfrac{N_2}{r_2}$
  - Distance travelled: $\theta_1 r_1 = \theta_2 r_2$
  - Combined: $\dfrac{r_1}{r_2} = \dfrac{N_1}{N_2} = \dfrac{T_1}{T_2} = \dfrac{\theta_2}{\theta_1} = \dfrac{\dot{\theta}_2}{\dot{\theta}_1} = \dfrac{\ddot{\theta}_2}{\ddot{\theta}_1}$
  - Parameters: N_1, N_2, r_1, r_2

Mechanical Systems

- Newton's 2nd Law for Translational Systems

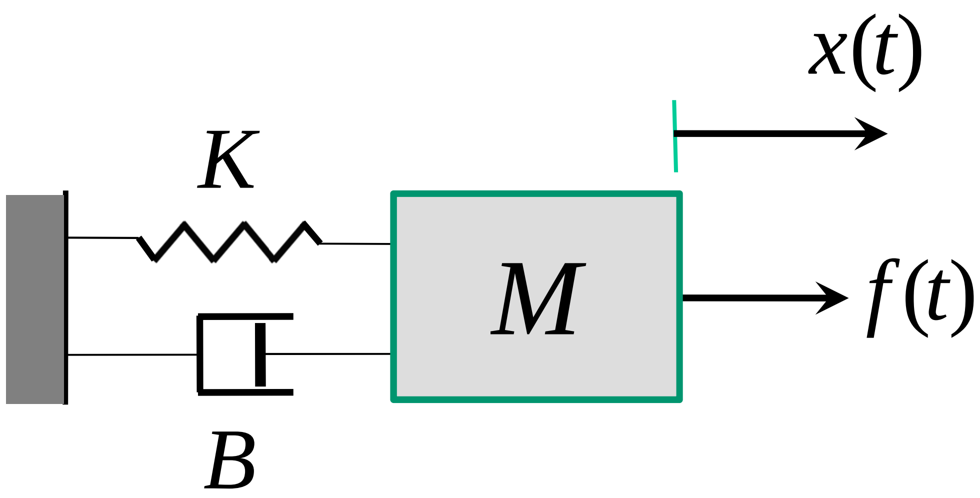
$$\sum_k F_k(t) = M\,\ddot{x}(t)$$

- Newton's 2nd Law for Rotational Systems

$$\sum_k T_k(t) = I\,\ddot{\theta}(t)$$

Examples

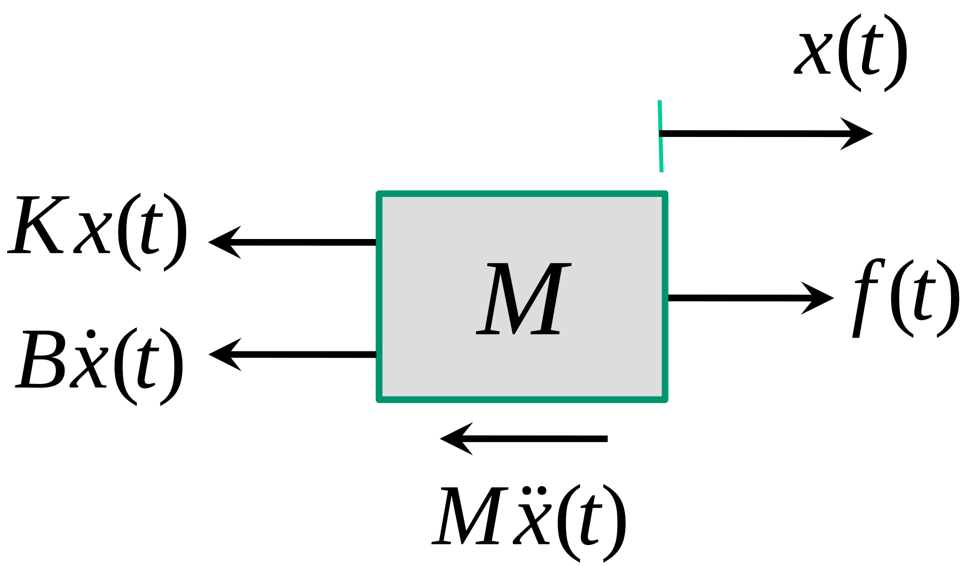
- Mass-Spring-Friction System



Examples

- Mass-Spring-Friction System

Free-body diagram



Examples

- Mass-Spring-Friction System

Newton's 2nd law

$$f(t) = M\ddot{x}(t) + B\dot{x}(t) + Kx(t)$$

$$\ddot{x}(t) = \frac{1}{M} f(t) - \frac{B}{M}\dot{x}(t) - \frac{K}{M}x(t)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

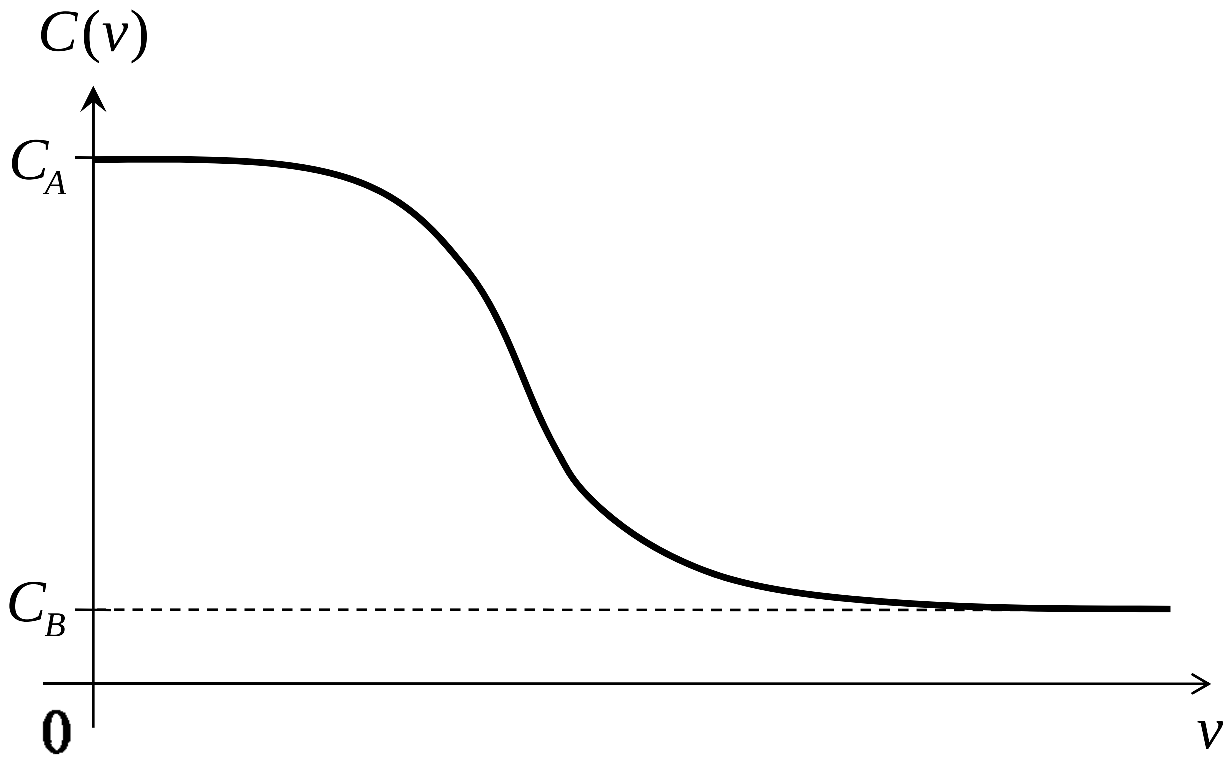
Examples

- Modelling a Soccer ball

Challenge: Reduce the ball to a mass, spring and damper.

Examples

- Modelling a Non-linear Capacitor



<u>Challenge</u>: Devise a circuit to model the above C - V characteristic.

Tutorial Exercises & Homework

- Tutorial Exercises
  - Burns, Examples 2.13 and 2.14
- Homework
  - Study Burns, Sections 2.1 to 2.5

Conclusion

- Mathematical Modelling
- Electrical Devices & Systems
- Mechanical Devices & Systems
- Some Examples
- Burns' Examples not covered (Self-study!)
- Thermal Systems & Fluid Systems not for Exam
- Tutorial Exercises & Homework
- Notation/Conventions – vary from Book to Book

Next Attraction! – Miss It & You'll Miss Out!

- Some Modelling Examples

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Thank you!

Any Questions?