CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 17)

Overview

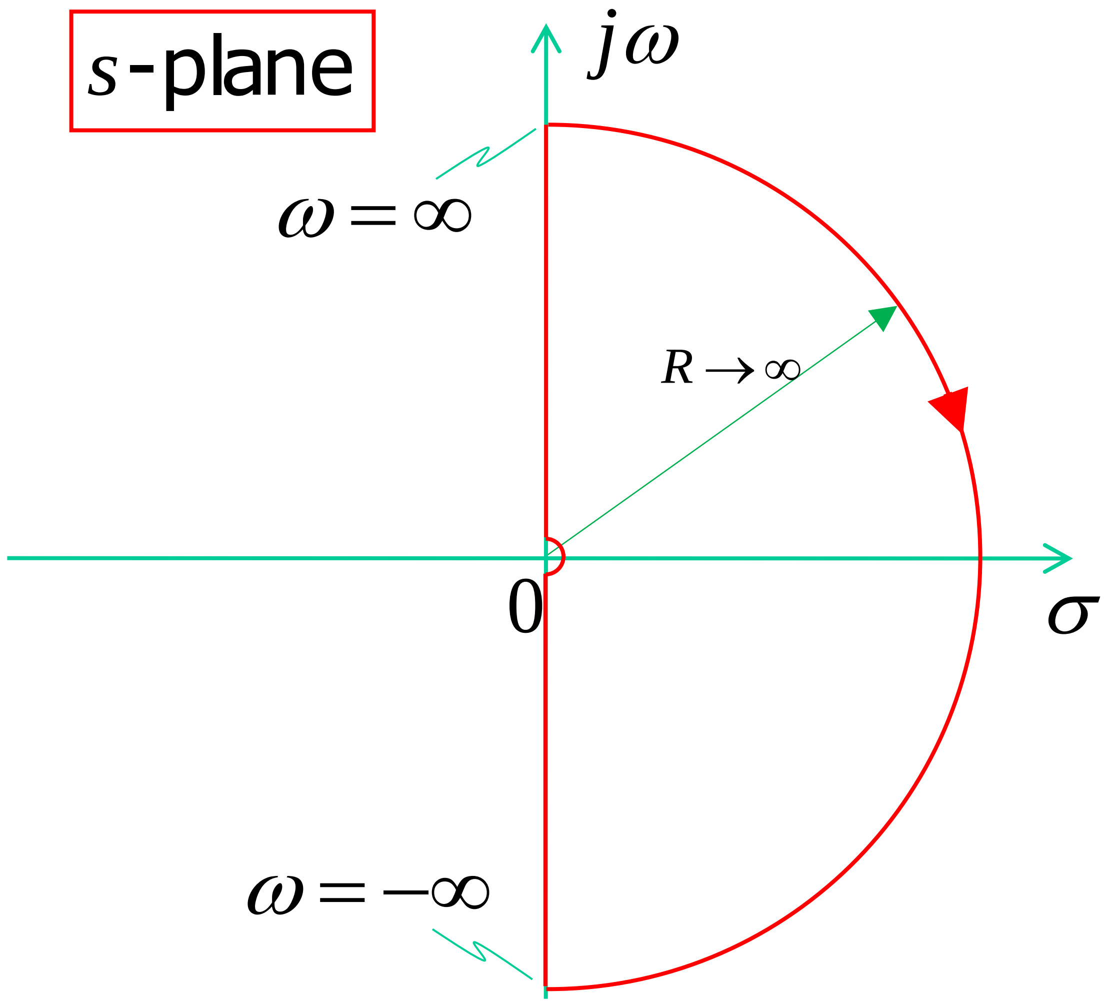
- First Things First!
- Nyquist Stability Criterion – Examples
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

- None

Nyquist Stability Criterion

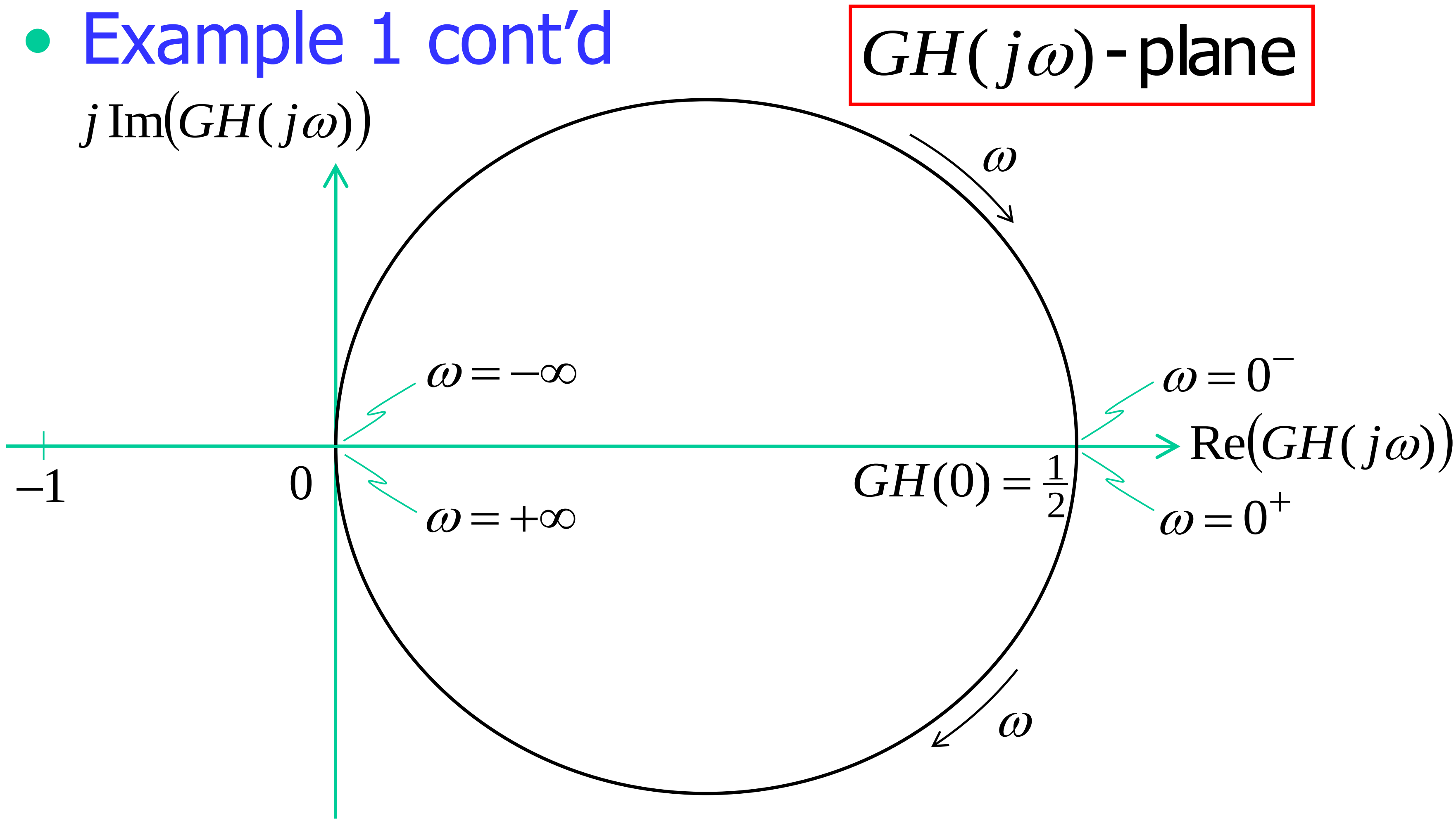


Nyquist Stability Criterion

- Example 1
  - First-order system, $G(s)H(s) = \dfrac{1}{s+2}$.
  - Open-loop poles: $s = -2$
  - No. of open-loop poles in the RHP: $P = 0$
  - To find N we plot the polar frequency response (*Nyquist plot*.)

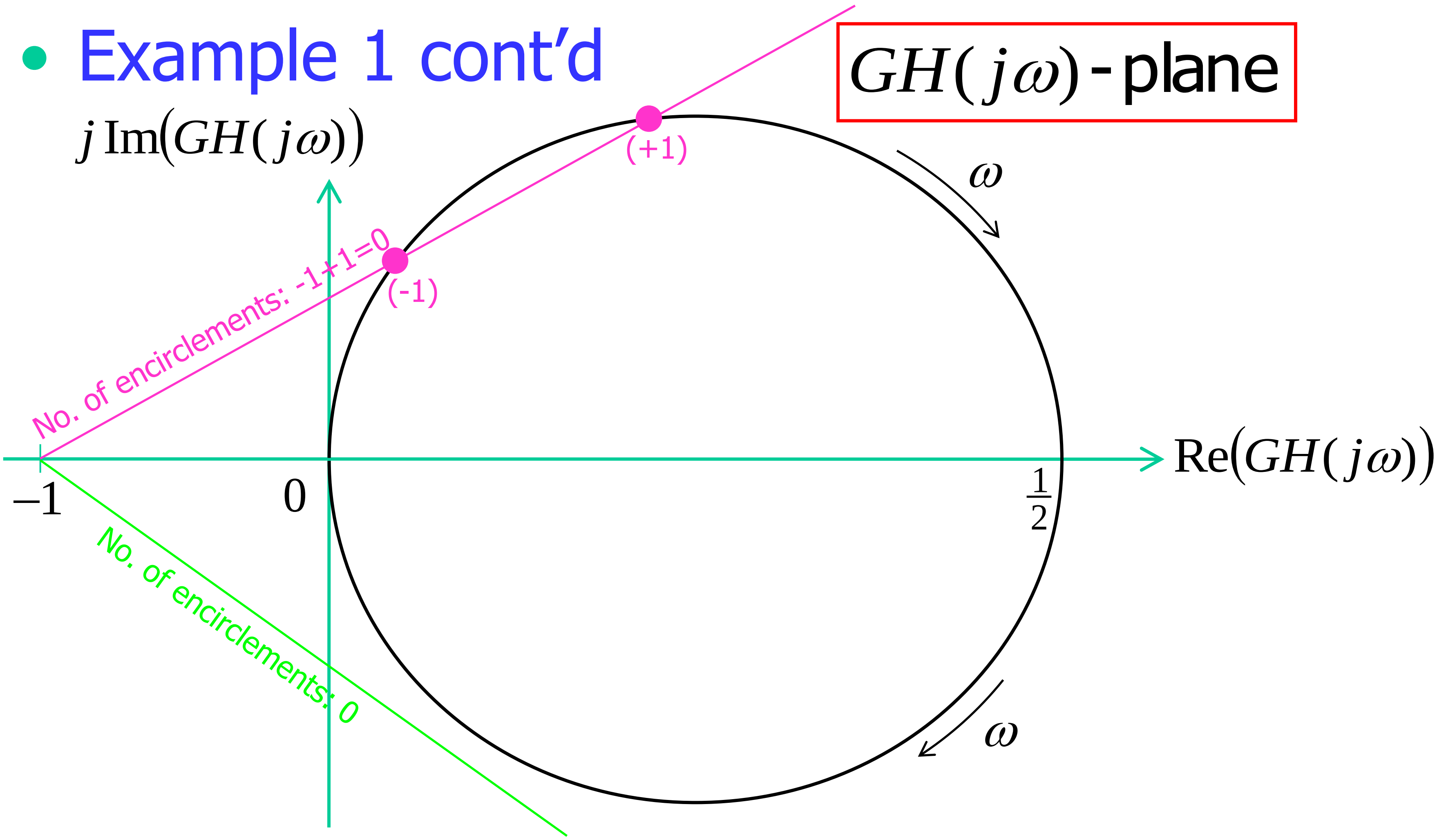
Frequency Response Representations

- Example 1 cont'd



Frequency Response Representations

- Example 1 cont'd



Nyquist Stability Criterion

- Example 1 cont'd
  - No. of encirclements: $N = 0$
  - Nyquist criterion:

    No. of closed-loop poles in RHP:

    $$Z = N + P = 0 + 0 = 0$$

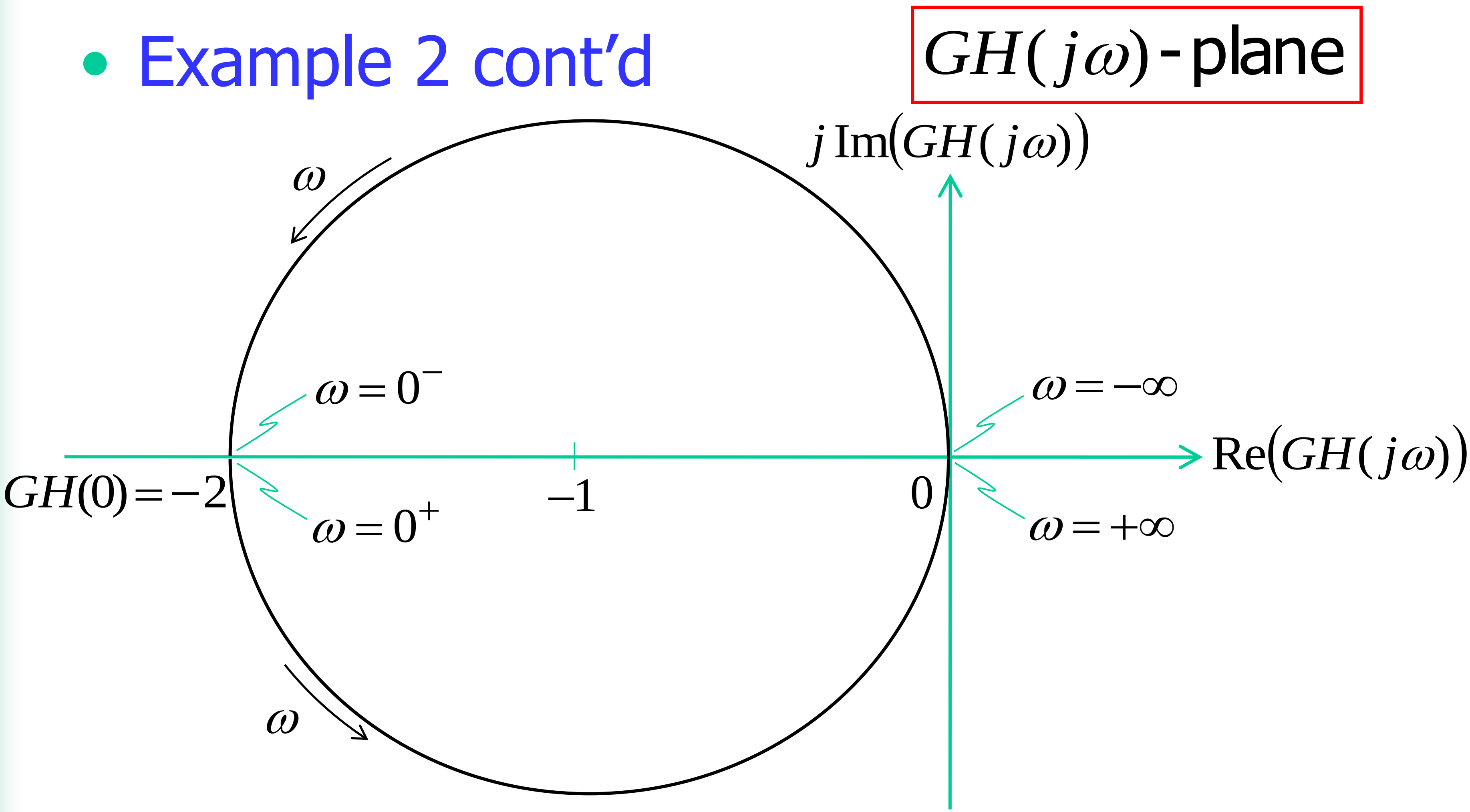
    Conclusion: The closed-loop system with characteristic equation $1 + G(s)H(s)$ is <u>stable</u>.

Nyquist Stability Criterion

- Example 2
  - First-order system, $G(s)H(s)=\dfrac{1}{s-\frac{1}{2}}$.
  - Open-loop poles: $s=\frac{1}{2}$ (unstable!)
  - No. of open-loop poles in the RHP: $P=1$
  - To find N we plot the Nyquist plot.

Frequency Response Representations

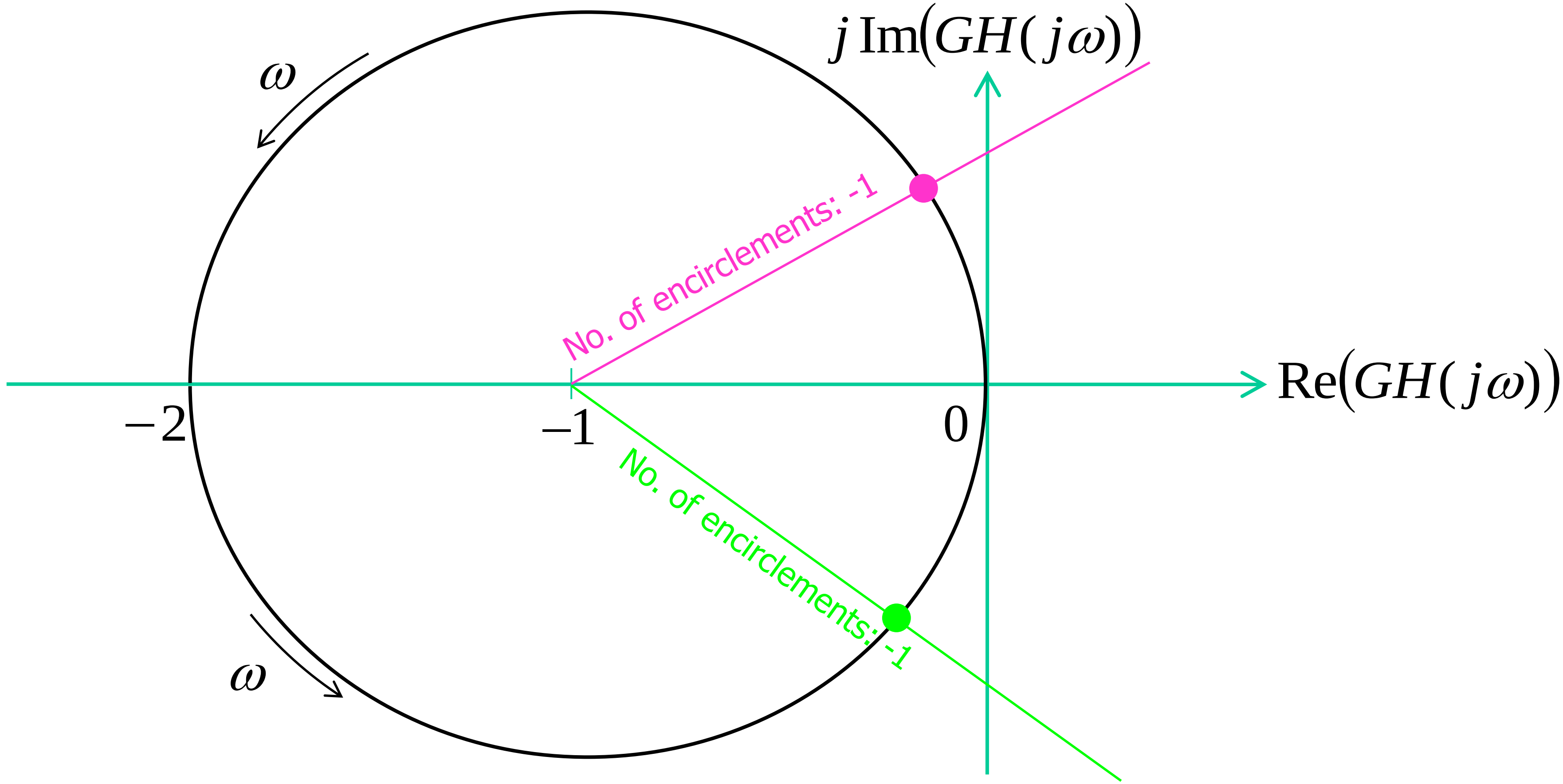
- Example 2 cont'd



Frequency Response Representations

- Example 2 cont'd

$GH(j\omega)$-plane



Nyquist Stability Criterion

- Example 2 cont'd
  - No. of encirclements: $N = -1$
  - Nyquist criterion:

    No. of closed-loop poles in RHP:

    $$Z = N + P = -1 + 1 = 0$$

    Conclusion: The closed-loop system with characteristic equation $1 + G(s)H(s)$ is <u>stable</u>.

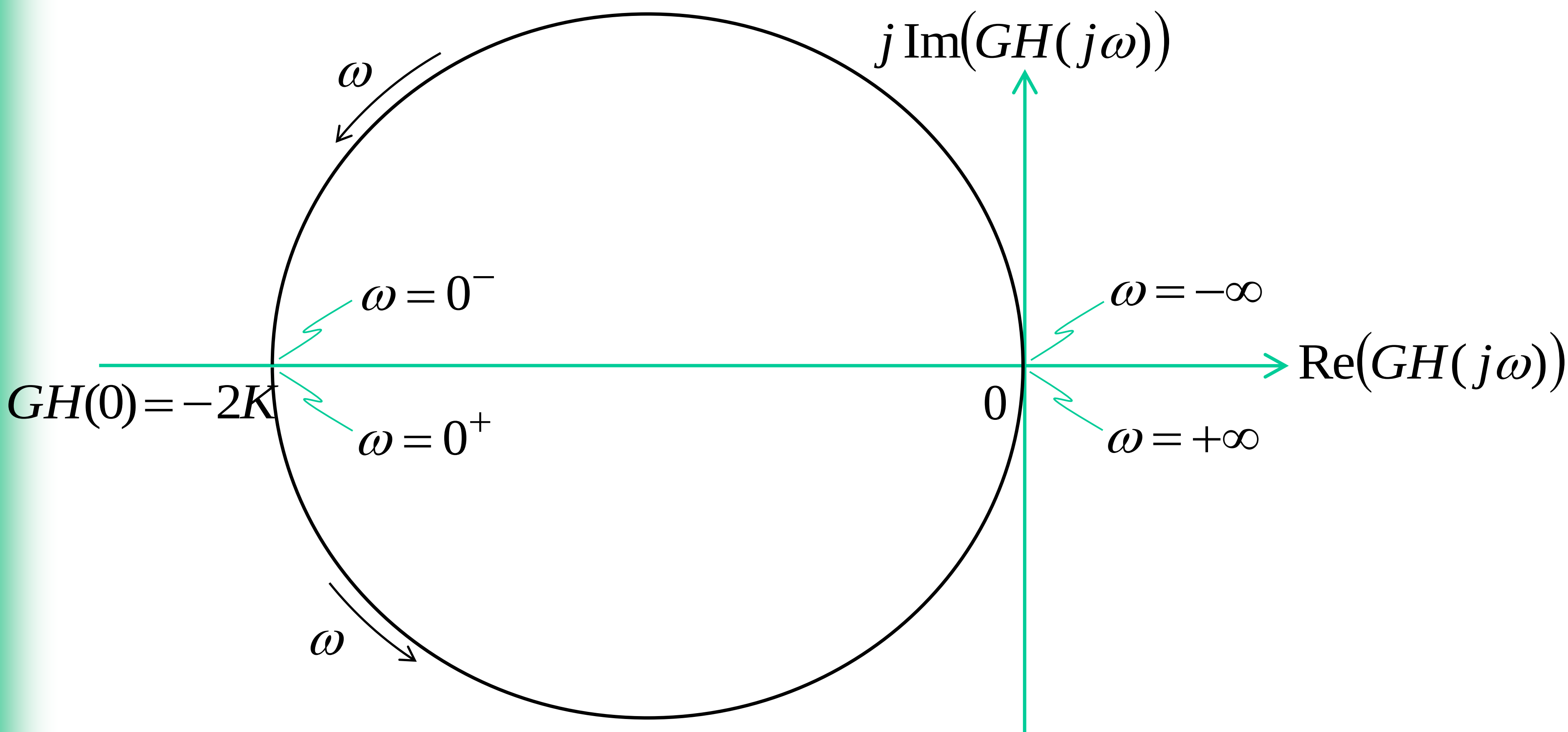
Nyquist Stability Criterion

- Example 3
  - First-order system, $G(s)H(s) = \dfrac{K}{s - \frac{1}{2}}, K > 0.$
  - Open-loop poles: $s = \frac{1}{2}$ (unstable!)
  - No. of open-loop poles in the RHP: $P = 1$
  - To find N we plot the Nyquist plot.

Frequency Response Representations

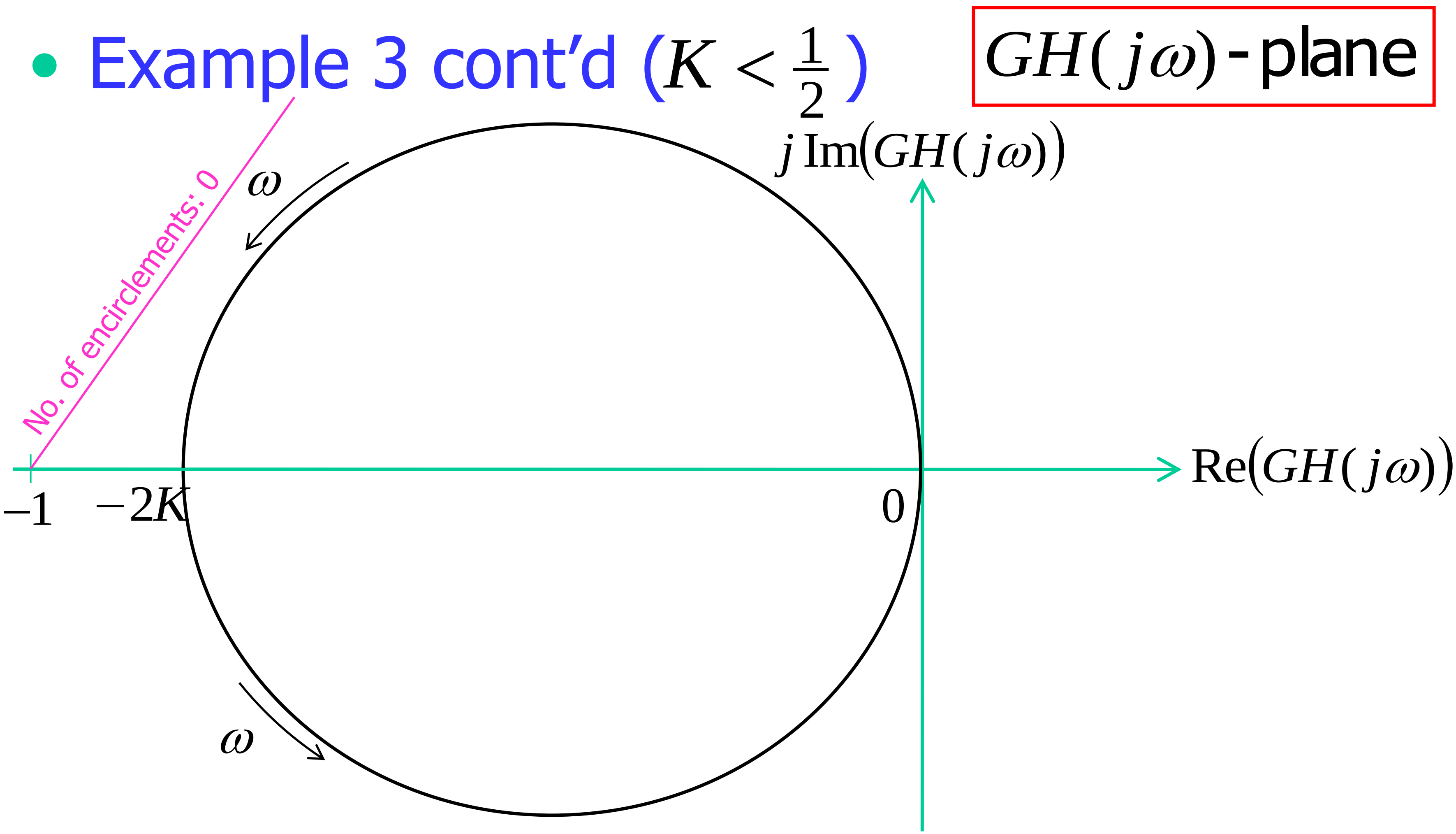
- Example 3 cont'd

$GH(j\omega)$ - plane



Frequency Response Representations

- Example 3 cont'd ($K < \frac{1}{2}$)



Nyquist Stability Criterion

- Example 3 cont'd ($K < \frac{1}{2}$)
  - No. of encirclements: $N = 0$
  - Nyquist criterion:

    No. of closed-loop poles in RHP:

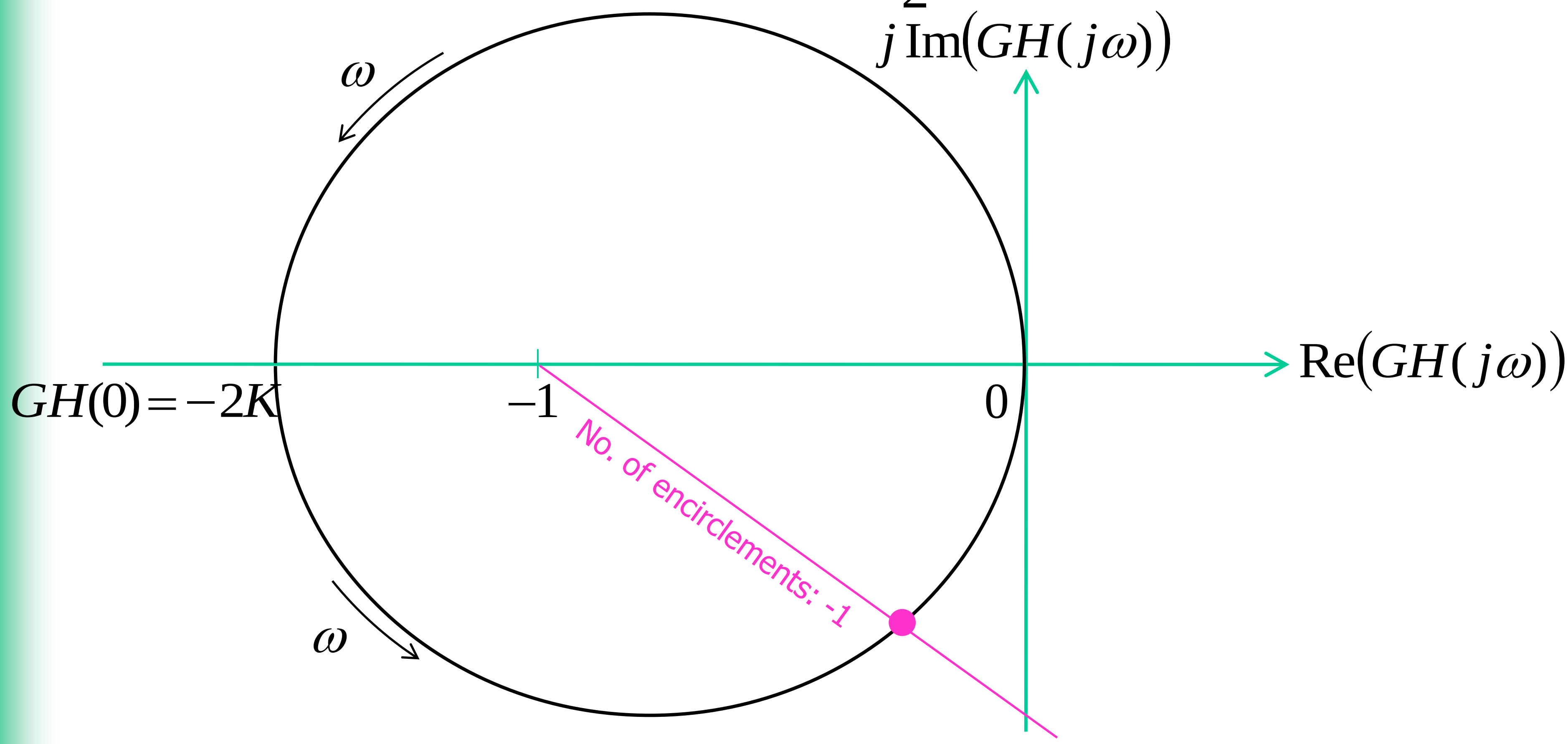
    $$Z = N + P = 0 + 1 = 1$$

    Conclusion: The closed-loop system with characteristic equation $1 + G(s)H(s)$ is <u>unstable</u>.

Frequency Response Representations

- Example 3 cont'd ($K > \frac{1}{2}$)

$GH(j\omega)$-plane



Nyquist Stability Criterion

- Example 3 cont'd ($K > \frac{1}{2}$)
  - No. of encirclements: $N = -1$
  - Nyquist criterion:

    No. of closed-loop poles in RHP:

    $$Z = N + P = -1 + 1 = 0$$

    Conclusion: The closed-loop system with characteristic equation $1 + G(s)H(s)$ is <u>stable</u>.

Nyquist Stability Criterion

- Example 3 cont'd ($K < 0$)
  - Since K is a (multiplicative) factor of the complex number $G(j\omega)H(j\omega)$, interpreted as a vector $G(j\omega)H(j\omega)\big|_{K<0}$ points in the direction of $-G(j\omega)H(j\omega)\big|_{K>0}$.
  - Conclusion: The plots $G(j\omega)H(j\omega)\big|_{K<0}$ and $G(j\omega)H(j\omega)\big|_{K>0}$ are *symmetric about the origin*.

Tutorial Exercises & Homework

- Tutorial Exercises
  - To be announced at the beginning of the tut session.

- Homework
  - Study all relevant sections in Burns.
  - Burns, Example 6.4,
  - Burns, Stability on the Bode diagram (pp. 170-171)

Conclusion

- Introductory Examples
- Burns, Example 6.4 (Self-study!)
- Burns, Stability on the Bode diagram (pp. 170-171) (Self-study!)
- Burns, Section 6.5 (Omit)
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Steady-State Error Analysis (Interlude)

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Thank you!

Any Questions?