CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 16)

Overview

- First Things First!
- Nyquist Stability Criterion
- Tutorial Exercises & Homework

- Next Attraction!

First Things First!

- Deadline for submitting your lab findings
  - Date agreed on: 19 October 2015

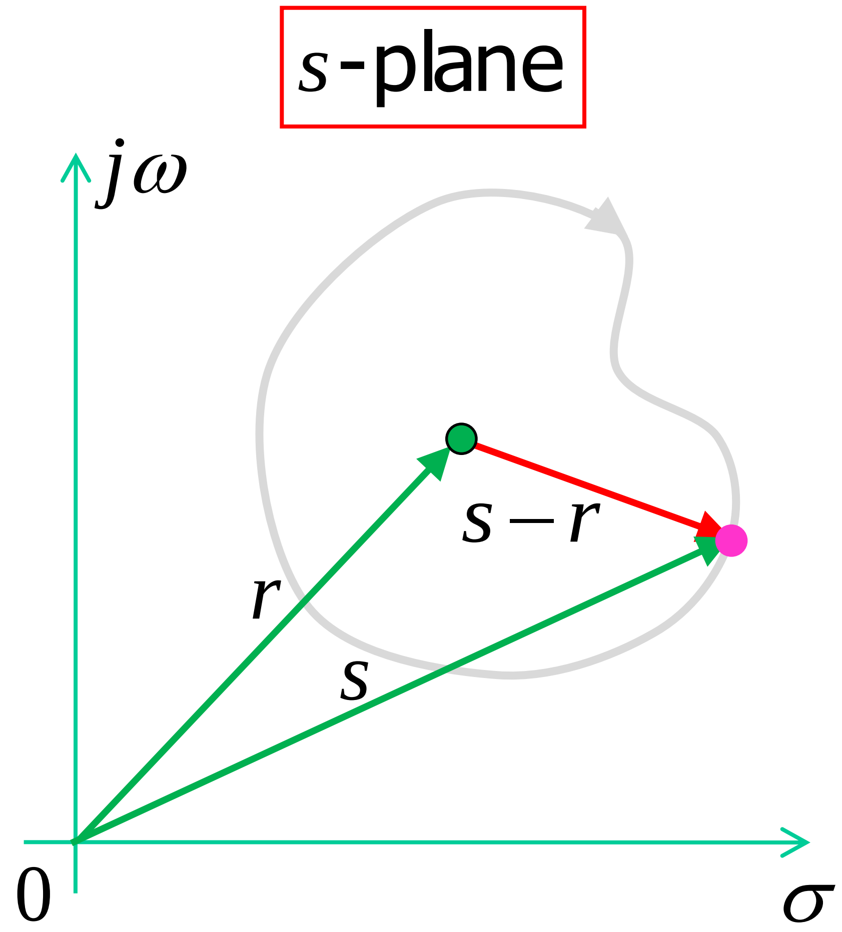
Cauchy's Principle of the Argument

– For the function $F(s) = s - r$, in the s-plane consider two closed contours; one enclosing r and one not enclosing r.

– Study the *net change in angle* of the vector $s - r$ as we traverse these contours starting at an arbitrary point going around once and back to this point.

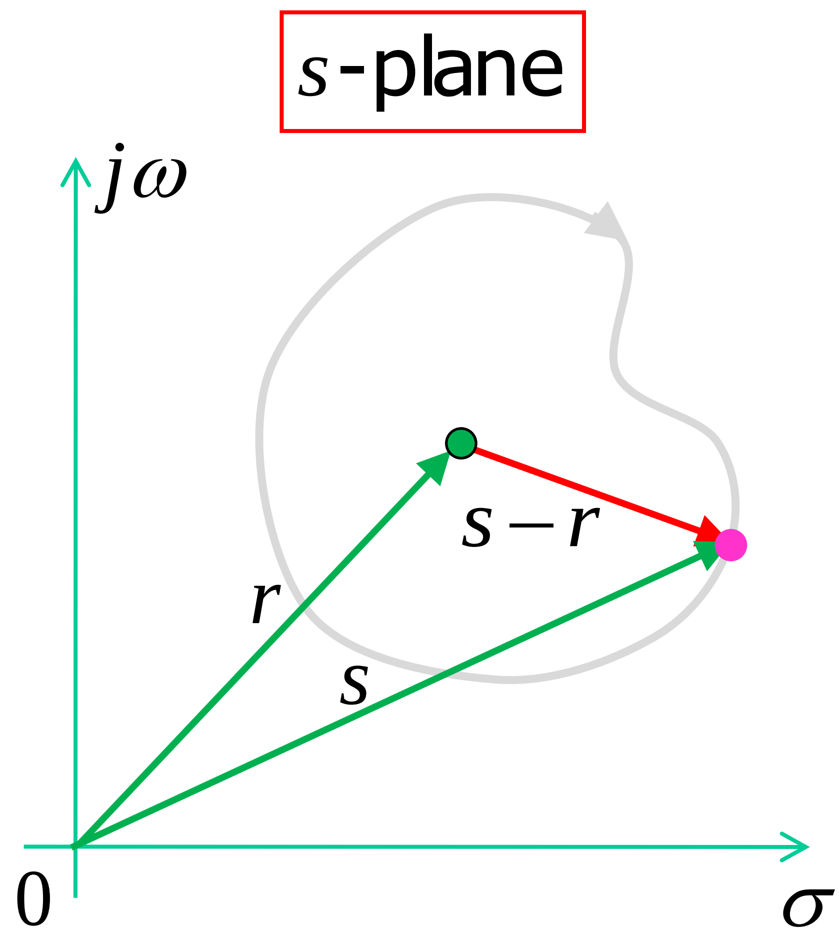
Cauchy's Principle of the Argument

- Enclosing Contour



Cauchy's Principle of the Argument

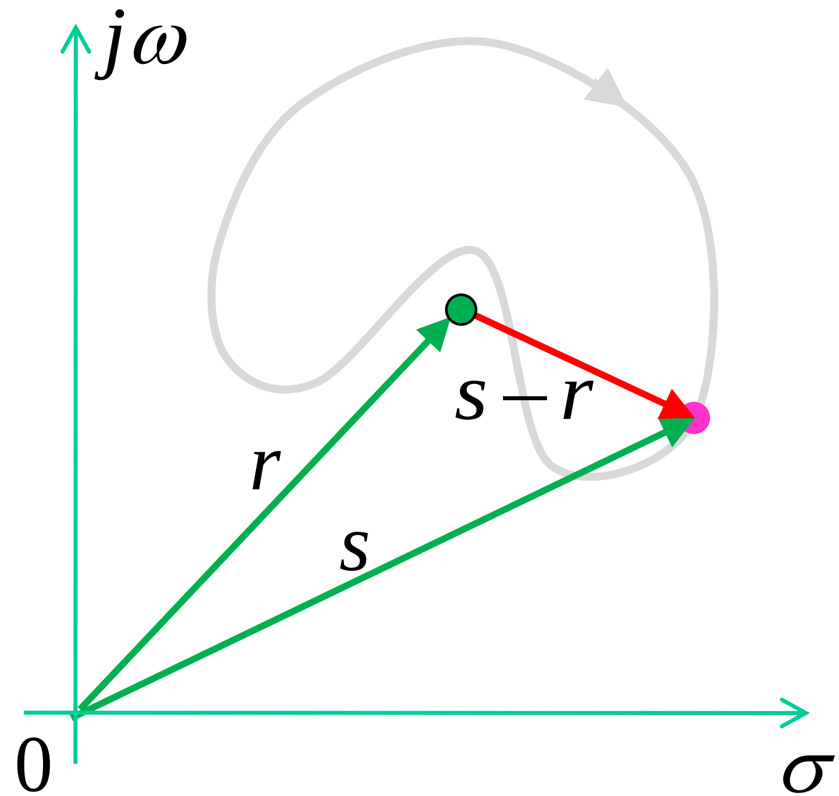
- Enclosing Contour
  - Each complete clockwise lap travelled <u>increases</u> the angle $\angle(s-r)$ by 2π.



Cauchy's Principle of the Argument

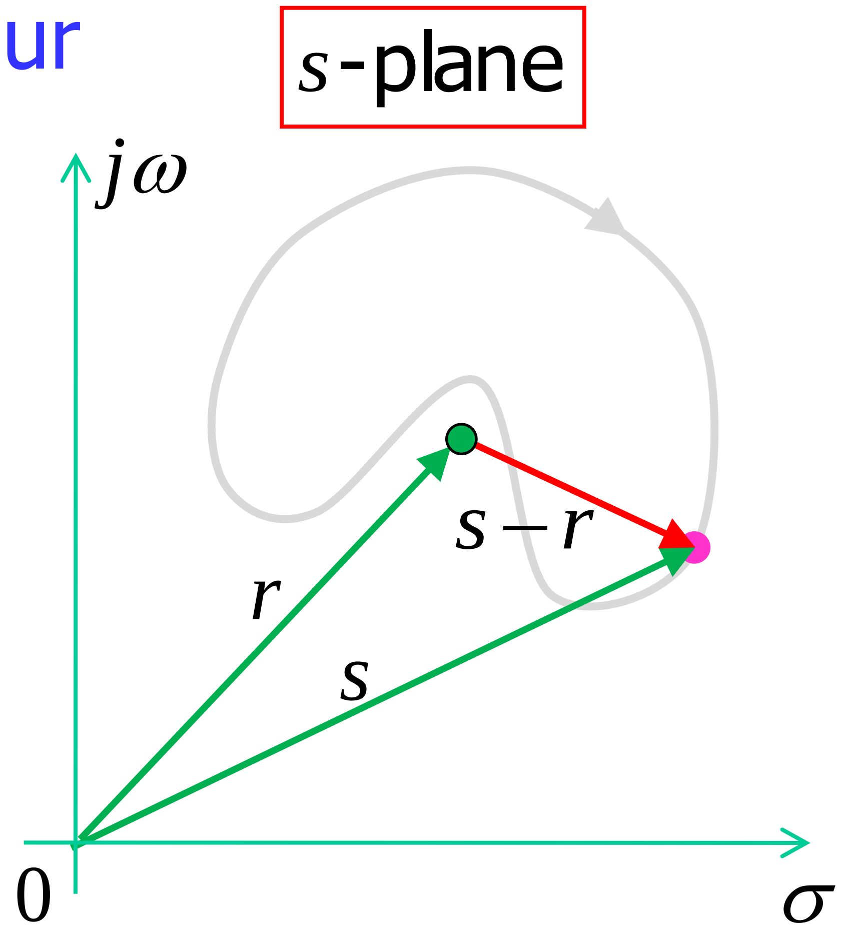
- Non-enclosing Contour

s-plane



Cauchy's Principle of the Argument

- Non-enclosing Contour
  - Now a complete lap does <u>not</u> change $\angle(s-r)$.



Cauchy's Principle of the Argument

- For the function $F(s)=\dfrac{1}{s-r}$, in the s-plane consider two closed contours; one enclosing r and one not enclosing r.
- Concluding about the net change in $\angle\dfrac{1}{s-r}$ is straightforward since $\angle\dfrac{1}{s-r}=-\angle(s-r)$.
- **Conclusion:** Here a lap along an enclosing contour <u>decreases</u> $\angle\dfrac{1}{s-r}$ by 2π while a non-enclosing contour yields no net change.

Cauchy's Principle of the Argument

– Now consider the function

$$F(s)=\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

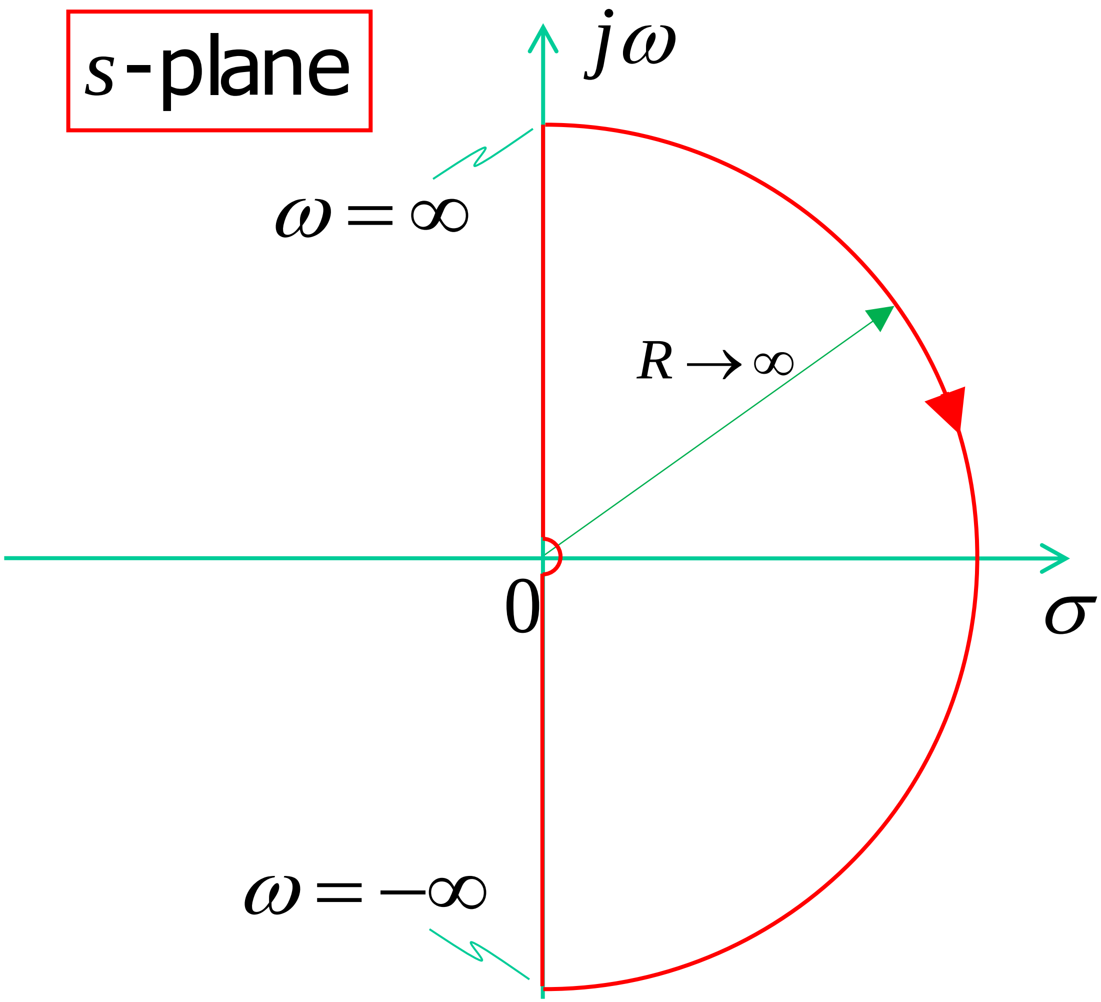
If a contour encloses Z zeros and P poles of $F(s)$, the net no. of encirclements of the origin of the $F(s)$-plane is $N=Z-P$.

Nyquist Stability Criterion

- Objective: Determine closed-loop stability.
- Choose the s-plane contour to be the semi-circle that encloses the RHP but excludes the LHP.
- Choose the complex function $F(s)$ to be the closed-loop characteristic polynomial,

$$F(s) = 1 + G(s)H(s) = 1 + \frac{N_G(s)N_H(s)}{D_G(s)D_H(s)}$$

Nyquist Stability Criterion



Nyquist Stability Criterion

– Closed-loop characteristic equation:

$$F(s)=1+GH(s)=1+\frac{N_G N_H(s)}{D_G D_H(s)}=0$$

$$\frac{D_G D_H(s)+N_G N_H(s)}{D_G D_H(s)}=0$$

– Zeros of $F(s)$ are all the closed-poles.

– Poles of $F(s)$ are all the open-loop poles.

Nyquist Stability Criterion

- Nyquist Criterion for $F(s) = 1 + G(s)H(s)$
  - With the contour enclosing the RHP <u>only</u>:

$$Z = N + P$$

P: No. of open-loop poles in the RHP.

N: Net no. of *clockwise* encirclements of the origin.

Z: No. of closed-loop poles in the RHP.

Question: What happens if we redefine $F(s) = G(s)H(s)$?

Nyquist Stability Criterion

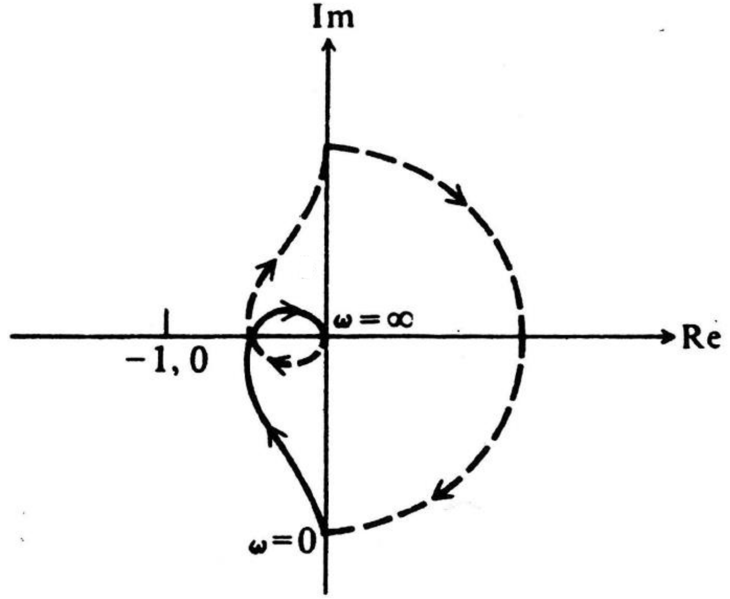
- Nyquist Criterion for $F(s) = G(s)H(s)$
  - With the contour enclosing the RHP <u>only</u>:

$$Z = N + P$$

- P: No. of open-loop poles in the RHP.
- N: Net no. of *clockwise* encirclements of the point -1.
- Z: No. of closed-loop poles in the RHP.

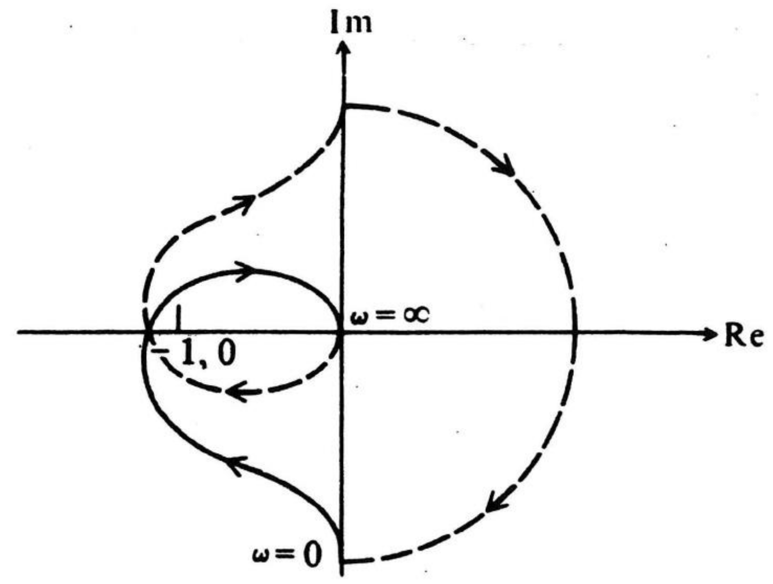
Tutorial Exercises & Homework

- Tutorial Exercises



(a) $G(s)H(s) = \dfrac{K_a}{s(1+T_1 s)(1+T_2 s)}$

$N = 0$
$P = 0$
$Z =$

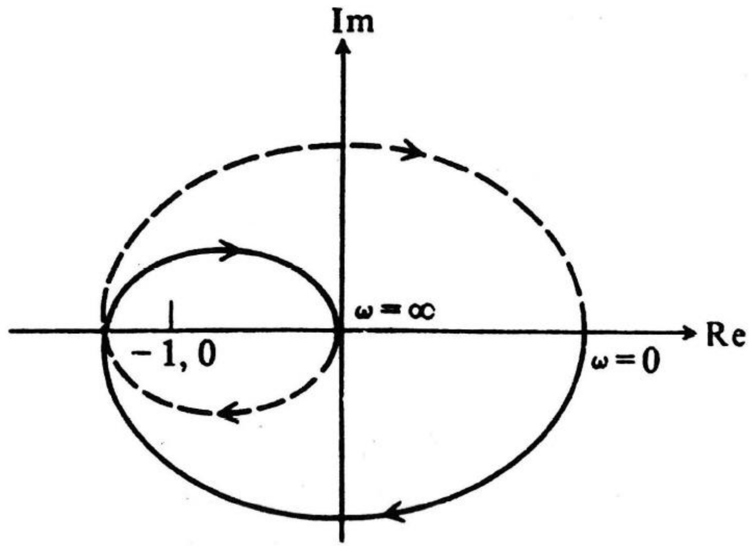


(b) $G(s)H(s) = \dfrac{K_b}{s(1+T_1 s)(1+T_2 s)}$

$N = 2$
$P = 0$
$Z =$

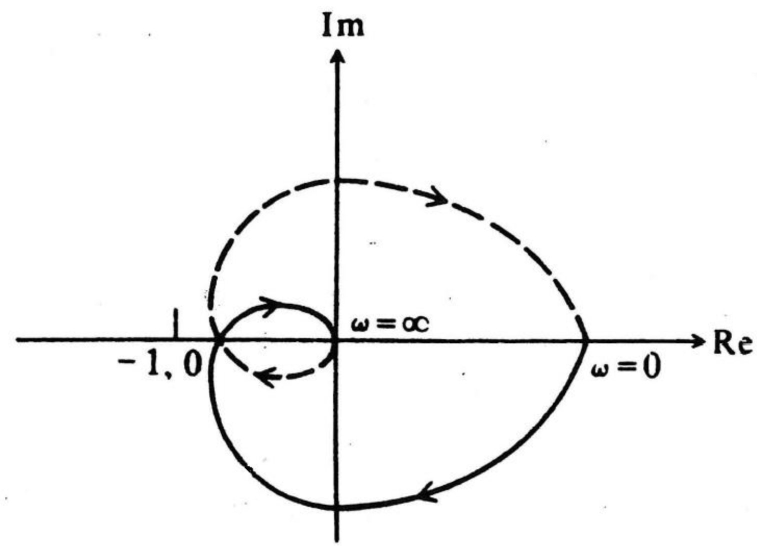
Tutorial Exercises & Homework

- Tutorial Exercises



(c) $G(s)H(s) = \dfrac{K_c}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$

$N = 2$
$P = 0$
$Z =$

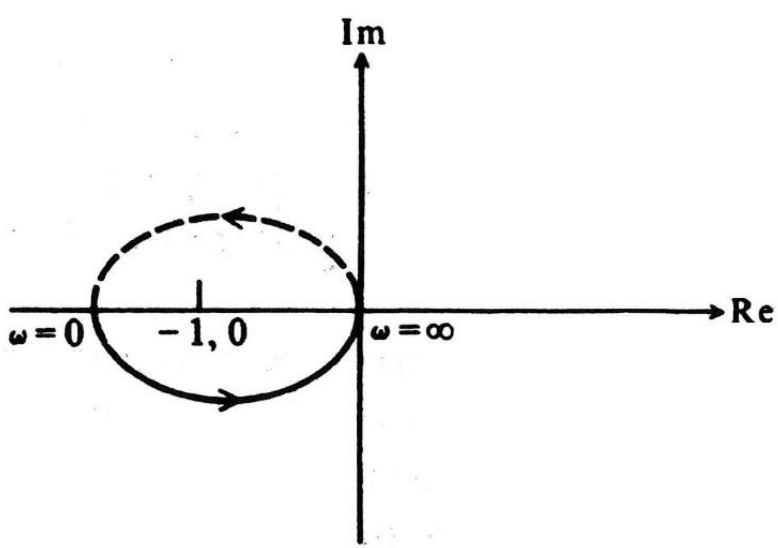


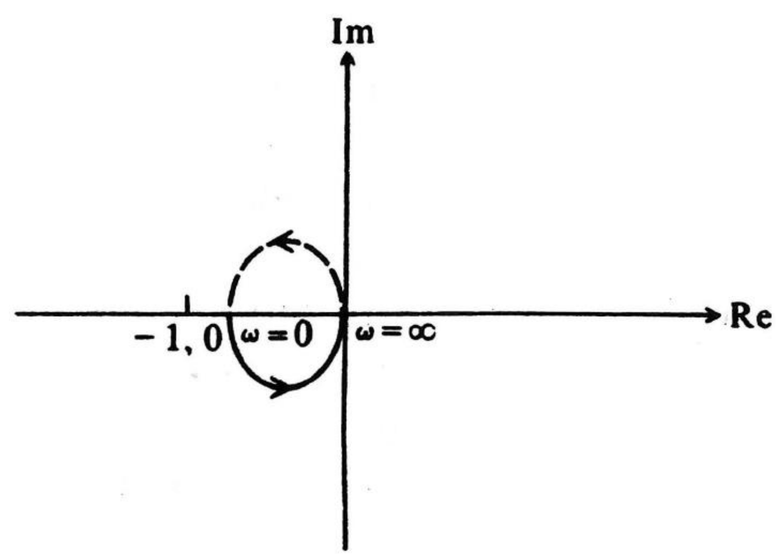
(d) $G(s)H(s) = \dfrac{K_d}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$

$N = 0$
$P = 0$
$Z =$

Tutorial Exercises & Homework

- Tutorial Exercises





(e) $G(s)H(s) = \dfrac{K_e}{(-1+Ts)}$

$N = -1$
$P = 1$
$Z =$

(f) $G(s)\,H(s) = \dfrac{K_f}{(-1+Ts)}$

$N = 0$
$P = 1$
$Z =$

Tutorial Exercises & Homework

- Homework
  - Study all relevant sections in Burns.

Conclusion

- Nyquist Stability Criterion
- Tutorial Exercises & Homework

Next Attraction! – Miss It & You'll Miss Out!

- Classical Design in the Frequency Domain Continued (Burns, Chapter 6)

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Thank you!

Any Questions?