CONTROL I

ELEN3016

Classical Design in the Frequency Domain

(Lecture 14)

Overview

- First Things First!
- Frequency Response Revised
- Graphical Interpretation of Frequency Response
- Tutorial Exercises & Homework

- Next Attraction!

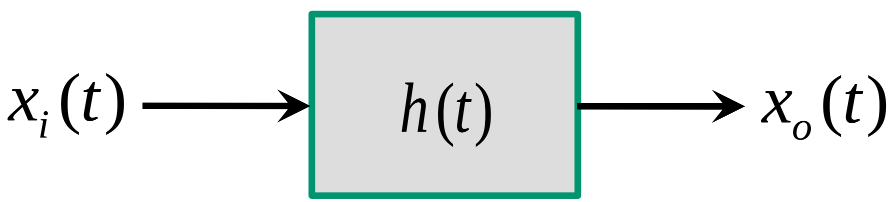
First Things First!

- Important Matters
  - Semester Test – Monday, 30th September 2015
    - Time: 8:00 – 10:00
    - Venue: FNBBA
    - Will cover Chapters 1-5
    - Closed-book test:

      All lectures, homework, tuts and variations of these.
    - One double-sided formula sheet with formulas but no diagrams!

Frequency Response

- Generalised Frequency Response



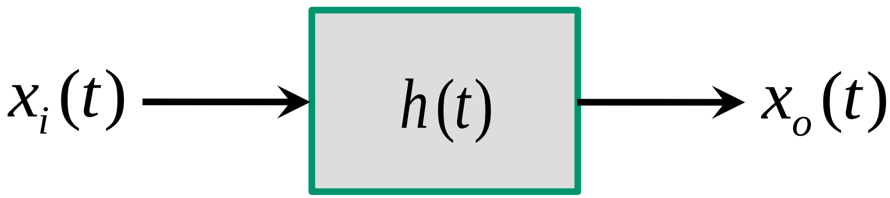
For $x_i(t) = e^{st}$ the forced response[1] is $x_o(t) = H_1(s)\, x_i(t)$ where $H_1(s) = \mathscr{L}\big(h(t)\big)$ is called the *generalised frequency response* and $s = \sigma + j\omega$ is called the *generalised frequency*.

The Laplace transform yields generalised frequency response.

1. BP Lathi, *Signals, Systems & Controls*, 1974, Sec. 2.16, Sec. 3.3 & Sec. 3.11.

Frequency Response

- Frequency Response



For $x_i(t) = e^{j\omega t}$ the forced response is $x_o(t) = H_2(\omega)\, x_i(t)$ where $H_2(\omega) = \mathfrak{F}\big(h(t)\big)$ is called the *frequency response* and ω is called the (radial) *frequency*.

The Fourier transform yields the frequency response.

Frequency Response

- Connection between GFR and FR?

$$H_2(\omega) = H_1(s)\big|_{s=j\omega} = H_1(j\omega)$$

The Fourier transform is the Laplace transform evaluated along the *j*ω axis in the *s*-plane.

We shall now discard the subscripts "1" and "2".

Frequency Response

- Graphical Interpretation

$$G(j\omega)H(j\omega) \equiv \left|G(j\omega)H(j\omega)\right| \angle G(j\omega)H(j\omega)$$

$$= \frac{\left(j\omega - z_1\right)\times\cdots\times\left(j\omega - z_m\right)}{\left(j\omega - p_1\right)\times\cdots\times\left(j\omega - p_n\right)}$$

$$\left|G(j\omega)H(j\omega)\right| = \frac{\left|j\omega - z_1\right|\times\cdots\times\left|j\omega - z_m\right|}{\left|j\omega - p_1\right|\times\cdots\times\left|j\omega - p_n\right|}$$

$$\angle G(j\omega)H(j\omega) = \sum_{i=1}^{m}\angle(j\omega - z_i) - \sum_{i=1}^{n}\angle(j\omega - p_i)$$

Frequency Response

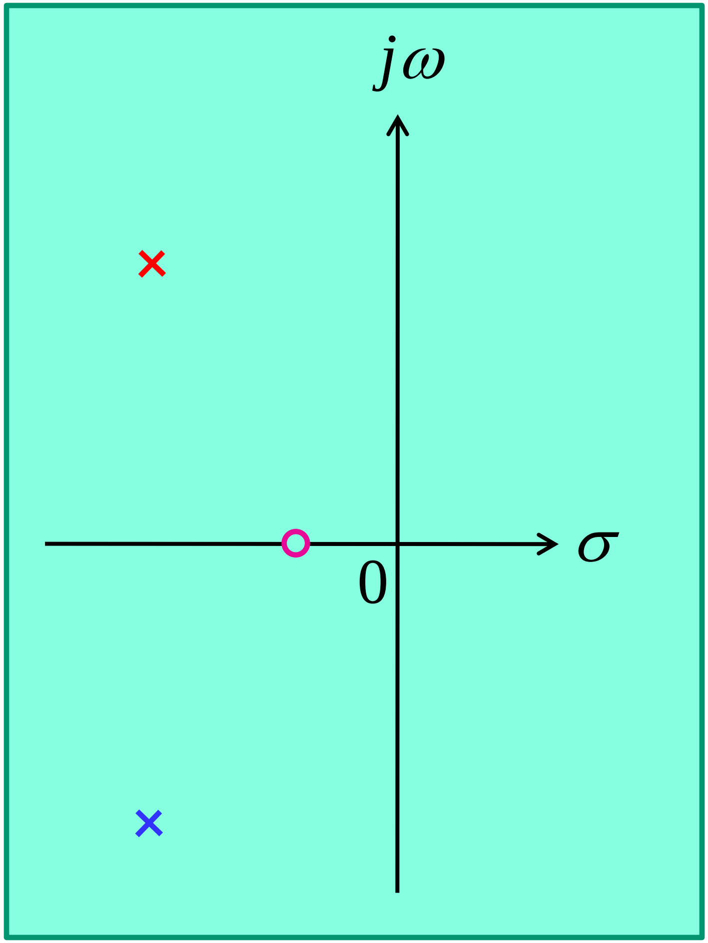
- Graphical Interpretation

Consider the system with loop transfer function

$$GH(s) \equiv G(s)H(s)$$

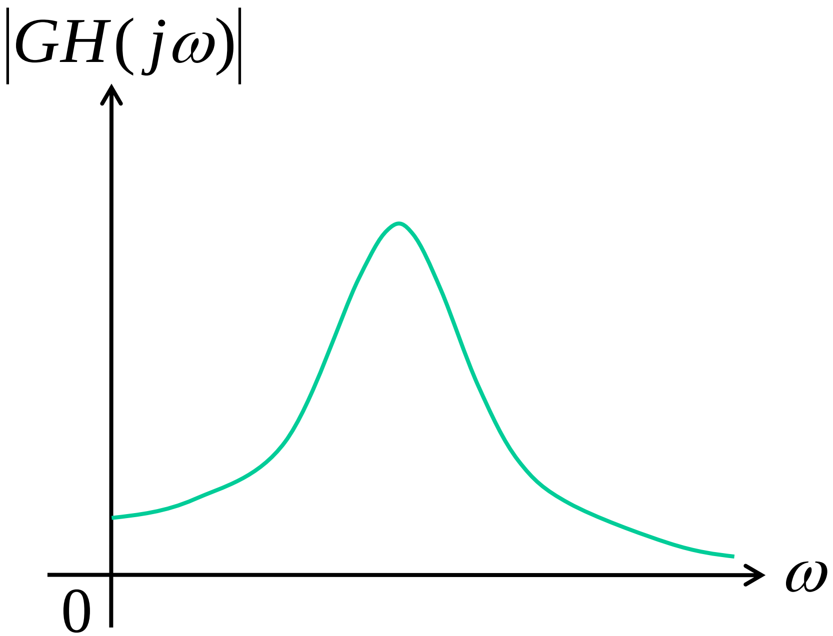
$$= \frac{(s+1)}{(s+2.5-j2.3)(s+2.5+j2.3)}$$

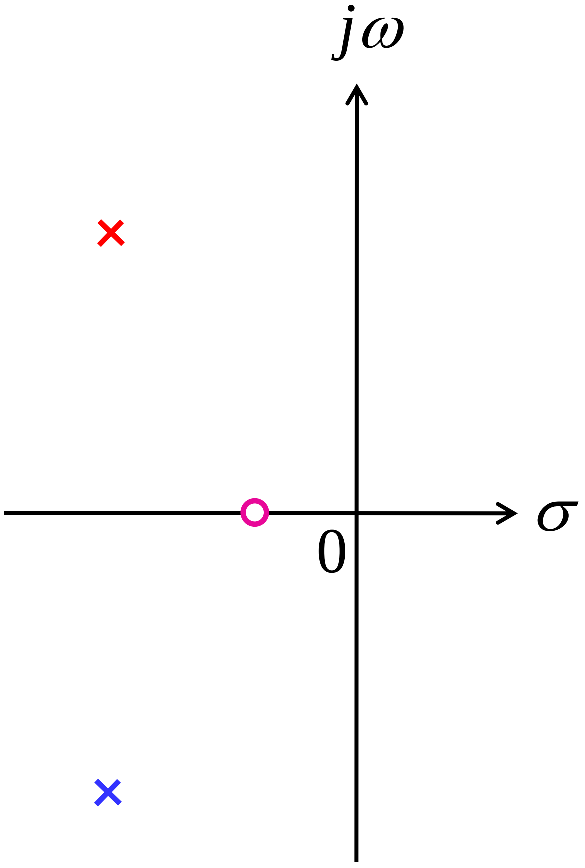
$$|GH(j\omega)| = \frac{|j\omega+1|}{|j\omega+2.5-j2.3||j\omega+2.5+j2.3|}$$



Frequency Response

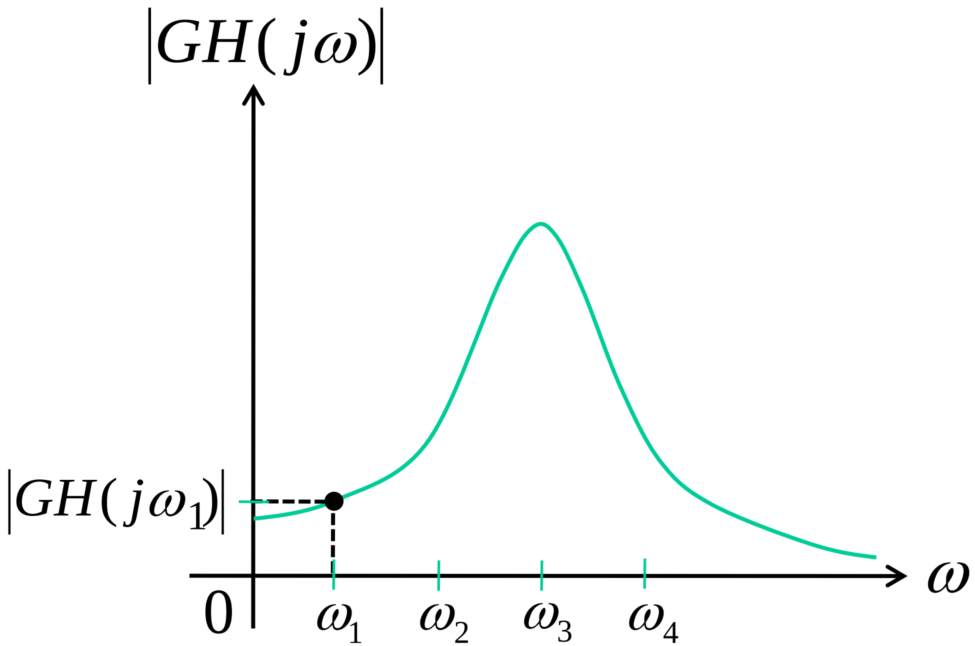
- Graphical Interpretation

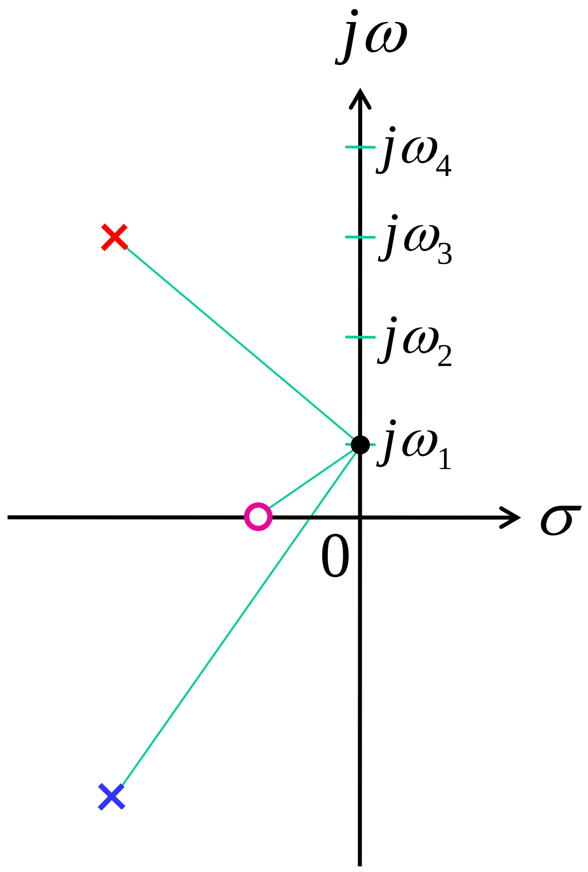




Frequency Response

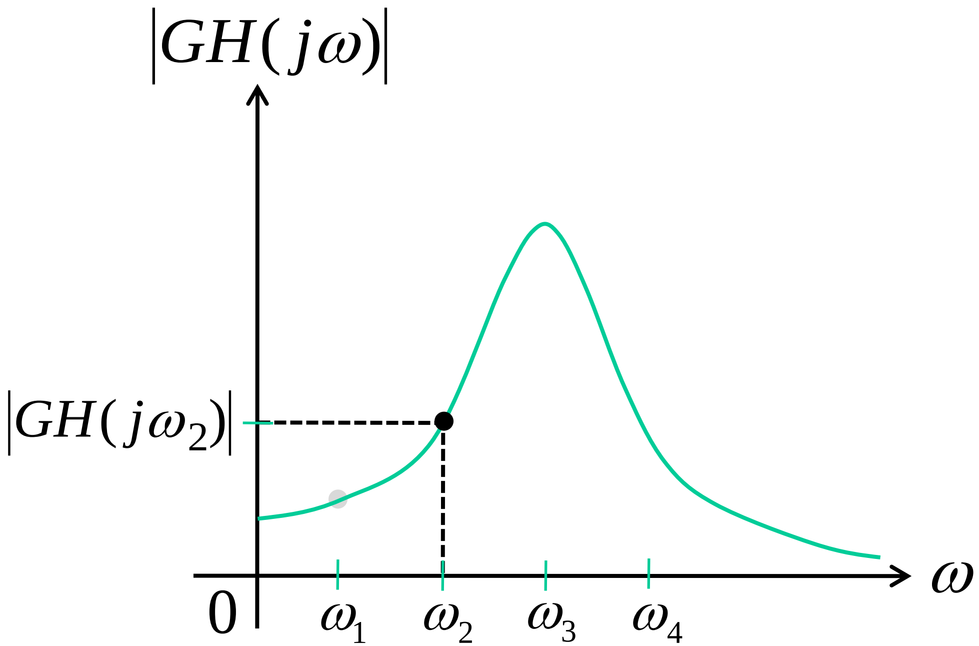
- Graphical Interpretation

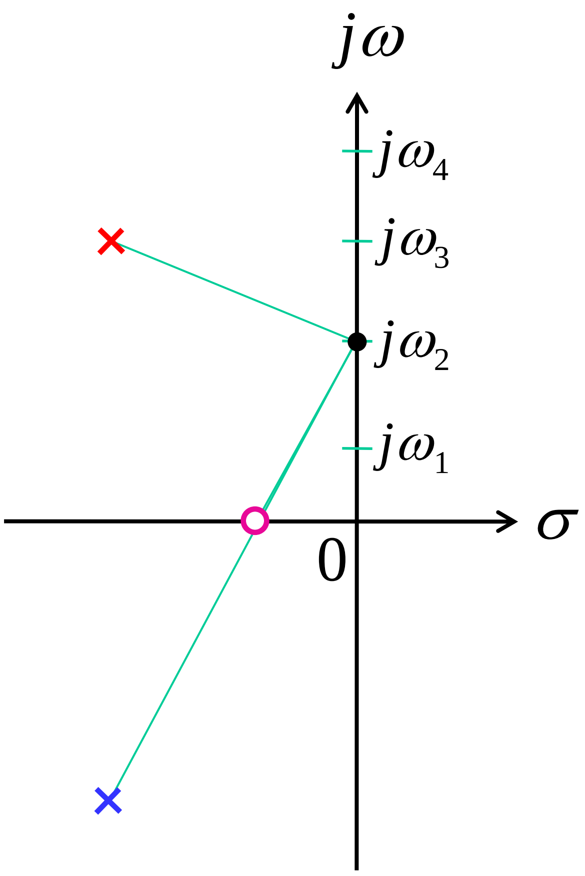




Frequency Response

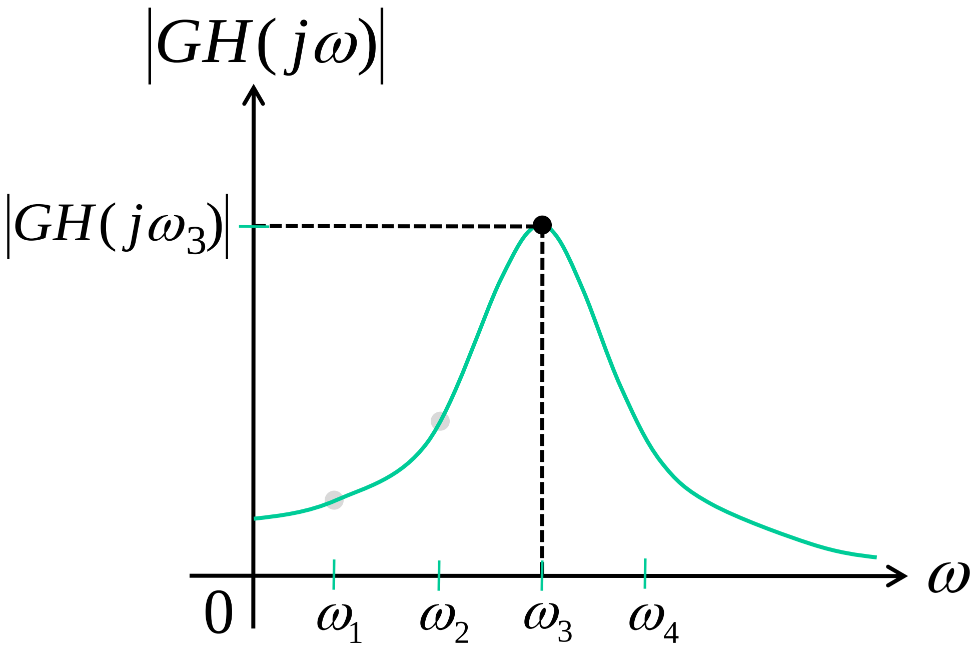
- Graphical Interpretation

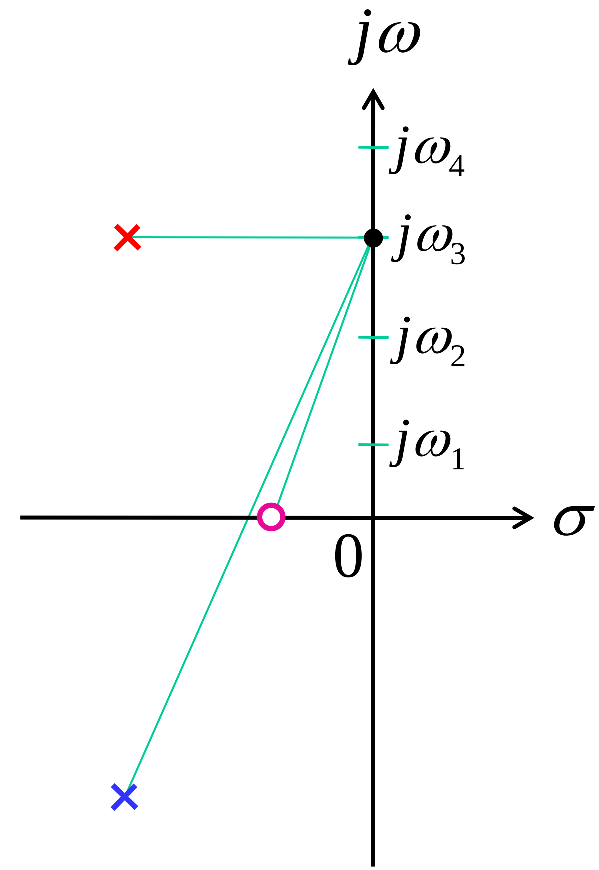




Frequency Response

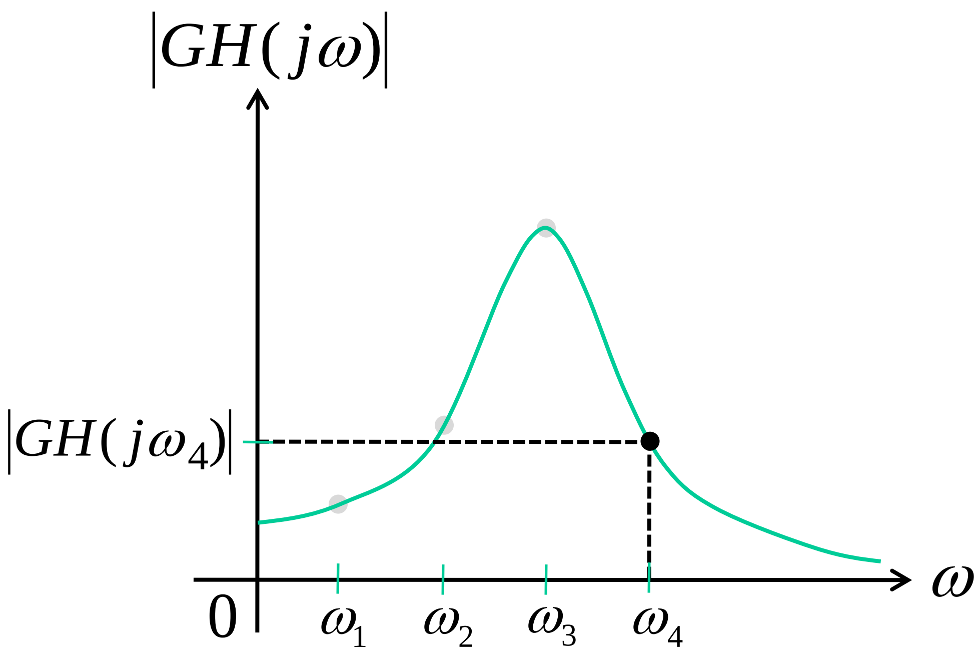
- Graphical Interpretation

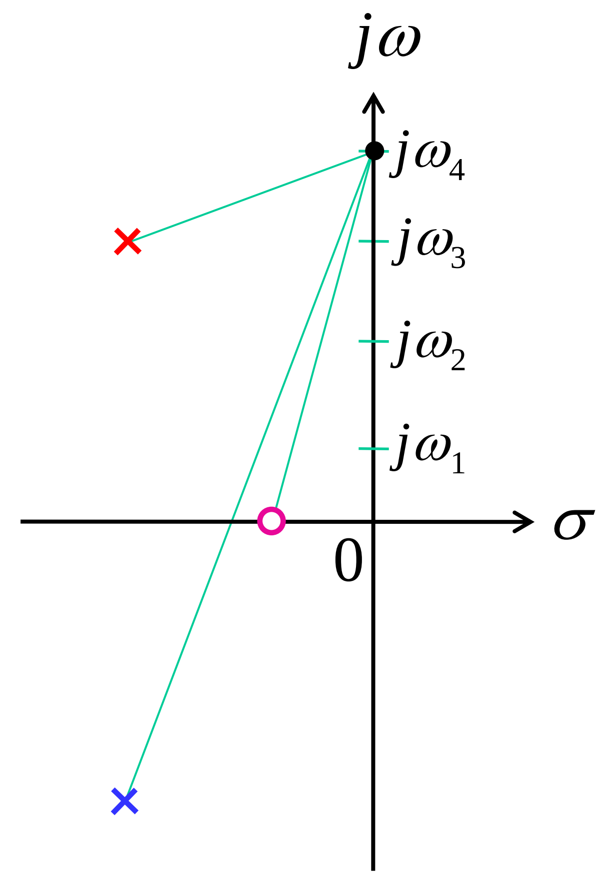




Frequency Response

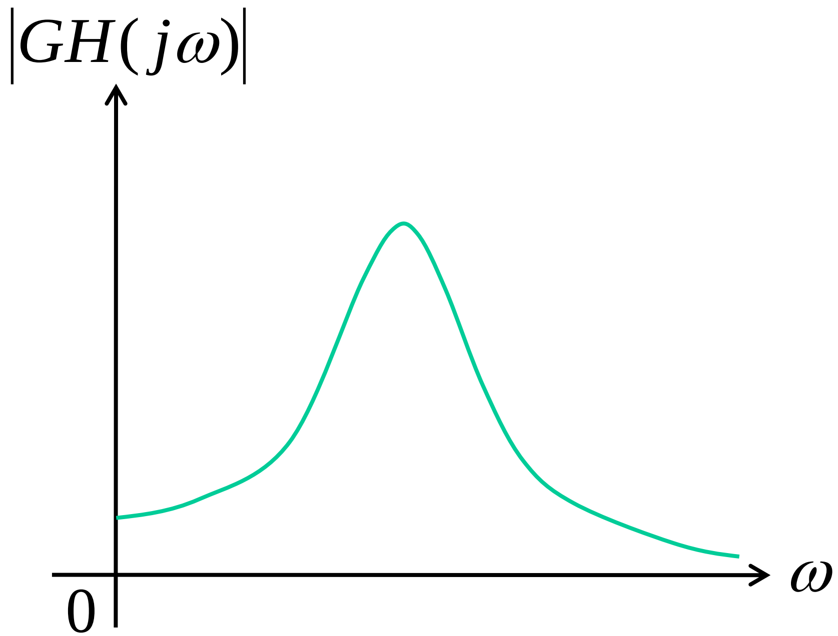
- Graphical Interpretation

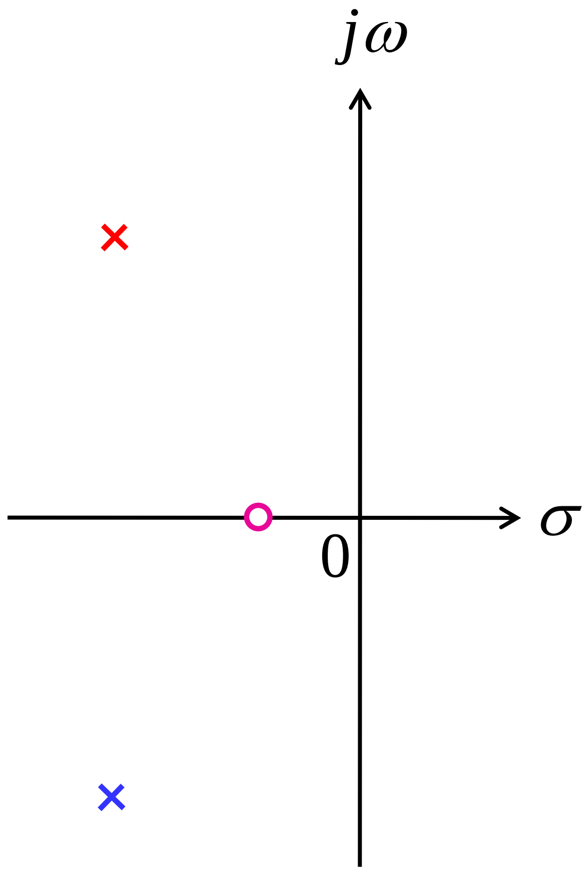




Frequency Response

- Graphical Interpretation





Tutorial Exercises & Homework

- Tutorial Exercises
  - Burns, Examples 6.10
  - Sketch the frequency response of $G(s) = \dfrac{1}{s+1}$.
- Homework
  - Study all relevant sections in Burns.

Conclusion

- Frequency Response
- Graphical Interpretation
- Burns, pp. 145-161 (Self-study!)
- Tutorial & Homework Exercises

Next Attraction! – Miss It & You'll Miss Out!

- Classical Design in the Frequency Domain Continued (Burns, Chapter 6)

...

Thank you!

Any Questions?