Tutorial 1 : Modelling

1) Consider a series resistor capacitor circuit(RC circuit R = 1Ω, C = 1F). Plot the capacitor voltage response when the input voltage is a unit step function. Note the capacitor voltage at times t = 1,2,3,4,5 and 6. How are the voltage values related to the half life of the system.

2) Consider the function $y(t)$ shown in Figure 1. Find the expression for this function.

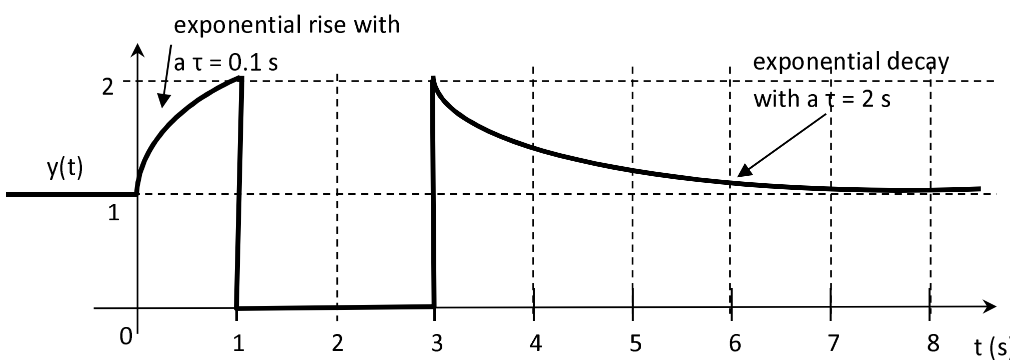

Fig. 1: question 2

3) Consider the circuit shown in Figure 2. The input to the circuit is the voltage $V_{in} = 12V$. The output of this circuit is the current inductor $i_L(t)$ and the switch is closed when t>0. Determine the current in the inductor,$i_L(t)$ for t¿0.

4) Consider the mechanical system shown in Figure 3 consisting of two 2 masses M_1 and M_2. Mass M_1 is in frictionless contact with the ground while the friction between the ground and mass M_2 is described by the damping coefficient B_2. The dashpot that attaches mass M_1 to the wall has damping coefficient of B_1. A time varying force $f(t)$ acts on mass M_1. Find the differential equations that relates x_1 and x_2 to the input force $f(t)$.

5) Figure 4 shows a system consisting of a mass-spring-damper system mounted on a massless cart. $u(t)$ is the dislacement of the cart and is the input to the system. The displacement $y(t)$ of the mass is the output. The displacements u and y are both measured relative to the ground.
   a) Find the differential equation that relates $u(t)$ to $y(t)$
   b) If now the cart has a mass M and a force $f(t)$ acts on the cart, find the differential equations that relate the input force $f(t)$ to the displacements $u(t)$ and $y(t)$

6) A DC motor drives an output shaft(inertia I_0 and damping coefficient B_0) via a gearbox with a reduction ratio of N as shown in Figure 5. The motor shaft has inertia I_m and damping coefficient B_m. The motor is excited by voltage $E_i(t)$(R_m = DC motor armature resistance, L_m = DC motor armature inductance, E_m = back emf, i_m = armature current)Find the differential equation that relates the input voltage $E_i(t)$ to the output shaft angle θ_o. Note θ_m is the angular displacement of the motor shaft.

7) Consider the quarter car model shown in Figure 6.
   - Develop a model for the active suspension system

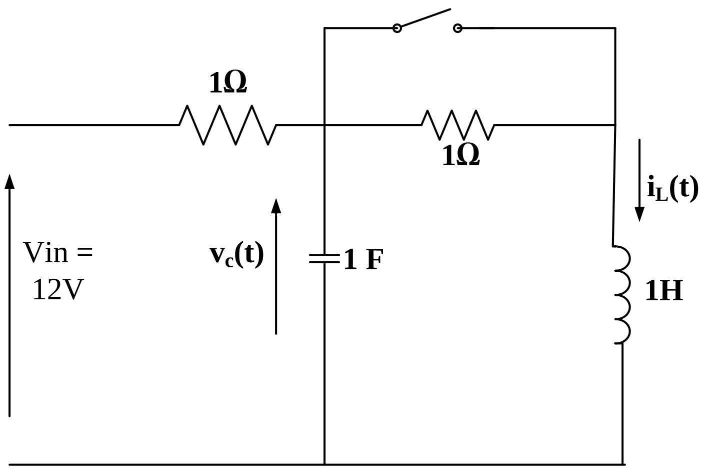

Fig. 2: question 3

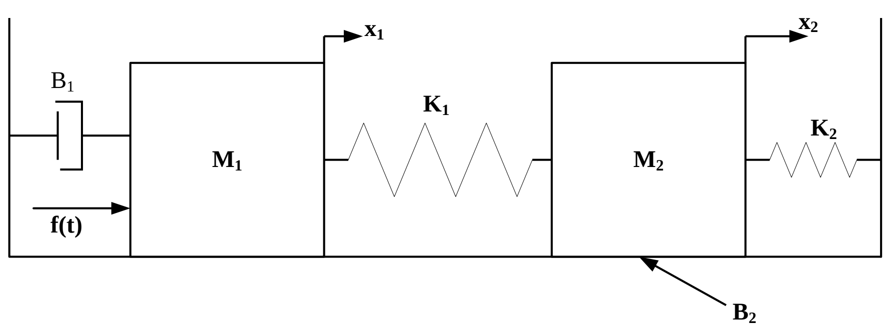

Fig. 3: question 4

- Develop a combined suspension and braking model

NOTE spring and damper are assumed to be linear.A slip based braking model is assumed(see class notes)

8) Figure 7 shows an electrical drilling machine. Taking the voltage $e_a\left(t\right)$ as the input, develop a mathematical model of the drilling operation.
9) A thermometer is used to measure temperature. At time t = 0 the thermometer is suddenly dipped from air at 20^0C to boiling water. 1 minute later the thermometer is suddenly transfered back to air. FInd the thermometer temperature at the following time instants 10s, 20s, 50s, 120s, and 300s.
   Mass = $5 \times 10^{-2}kg$
   Surface area = $10^{-3}m^2$
   Specific heat = $0.2Jkg^{-1o}C^{-1}$
   Heat transfer coefficient for air = $0.2Wm^{-2o}C^{-1}$
   Heat transfer coeeficient for water = $1.0Wm^{-2o}C^{-1}$

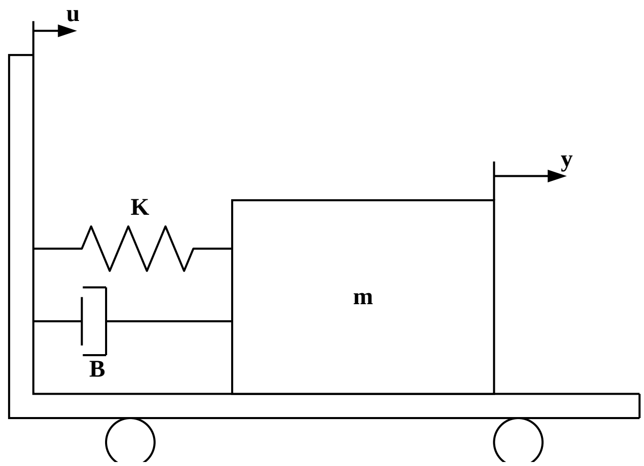

Fig. 4: question 5

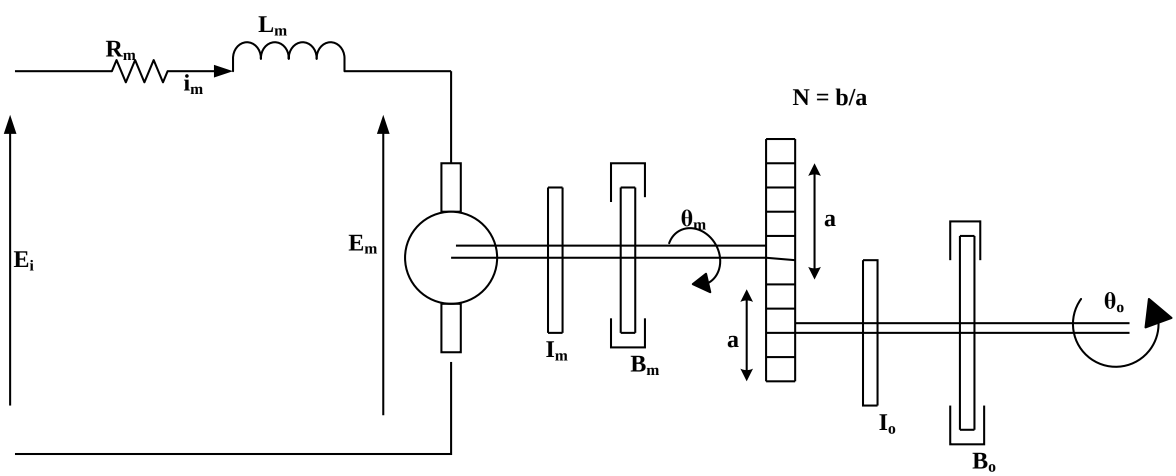

Fig. 5: question 6

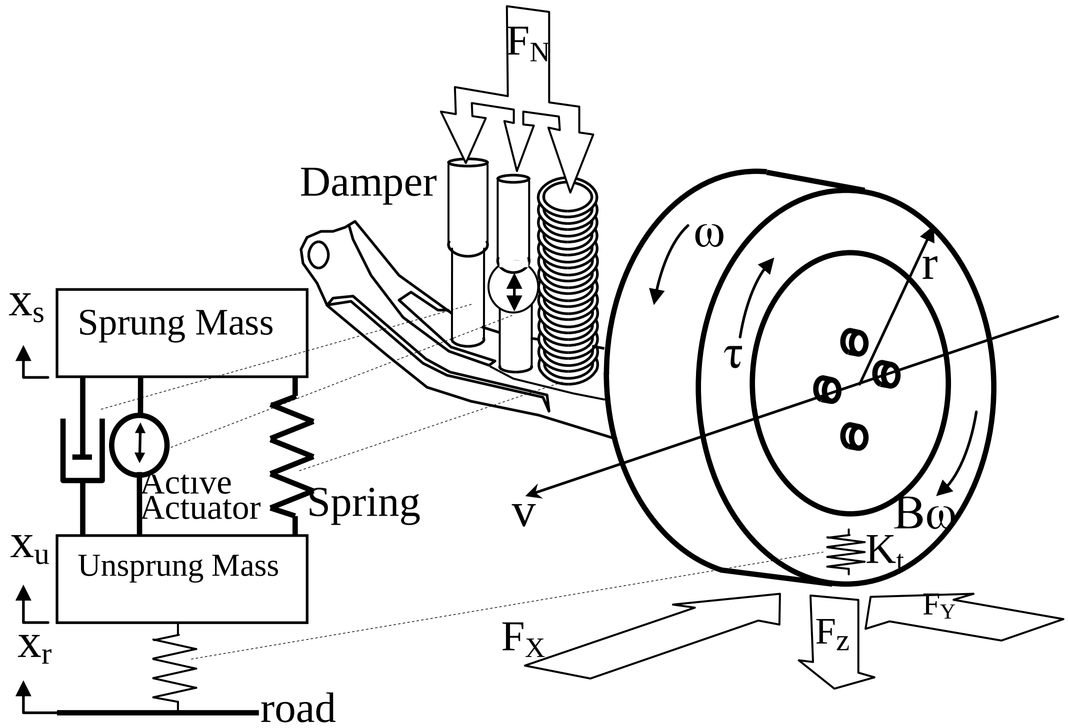

Fig. 6: question 7

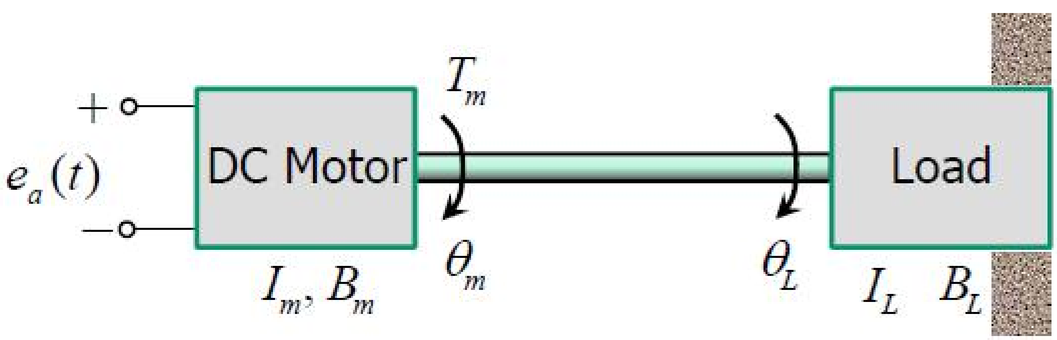

Fig. 7: question 8