Concept Definitions and Maps for Electric Circuits

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Introduction

The paper *A Conceptual Framework for Electric Circuits* [Ref] provides a set of definition of the concepts and maps these to answer the *focus question*:

> What is circuit theory and analysis that serves as a foundation for much of electrical engineering?

for the case of resistive circuits, that is, circuits containing sources and resistors.

Assumed prior knowledge from Physics is defined in the paper and is supplemented under the definitions in response to Q6 below.

The focus question is addressed via eight subsidiary questions, each corresponding to a group of related concepts. The first five questions are addressed in the paper and the concept definitions and concept maps are repeated below:

- Q1: What are the physical origins of electric circuit variables and elements?
- Q2: What are the circuit elements and variables and what is their behavior?
- Q3: What are the fundamental laws and principles of circuit analysis?
- Q4: What are the principles and methods of analysis of a circuit as a system by simplifying parts or the whole circuit?
- Q5: What are the principles and methods of analysis of a circuit as a system by systematically setting up equations for solution?

The remaining three questions are addressed in this document, namely:

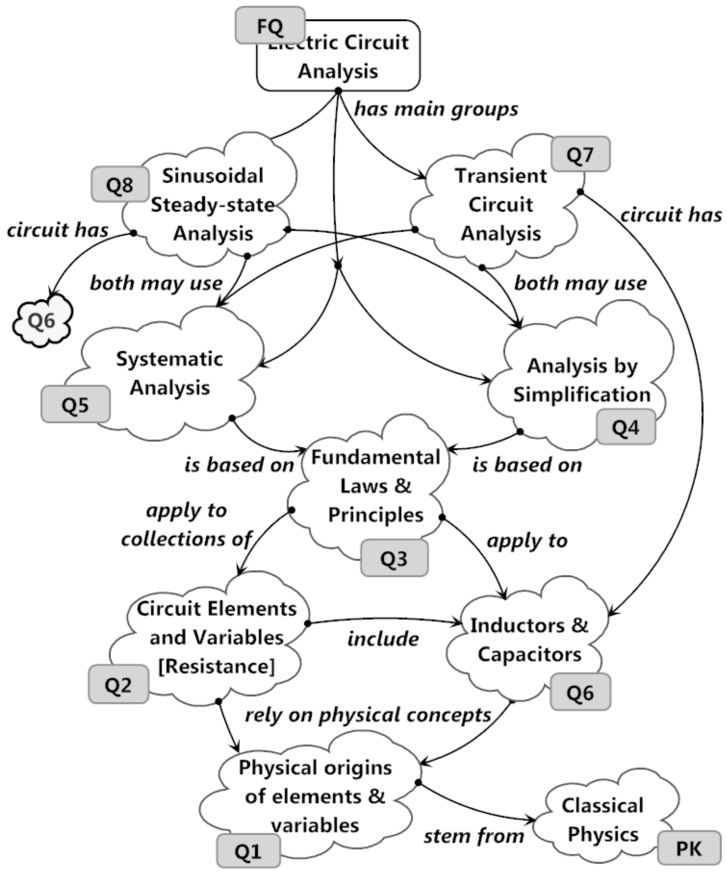
- Q6: What are the physical basis and behavior as circuit elements of (a) capacitors and (b) inductors?
- Q7: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors with time-varying sources?
- Q8: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors under sinusoidal steady state conditions?

The figure below maps the groups of concepts that emerge in response to these questions. Such groups of concepts are referred to as *segments* of the knowledge base.

An estimate of the size of the knowledge base is given in terms of the number of concepts and links between concepts.

This navigation bar is placed at the end of each question:

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8



Repeat of Figure 2 in paper: High level concept map addressing the focus question

Question 1: Physical origins of electric circuit variables and elements

Table II: Concepts for Q1: What are the physical origins of electric circuit variables and elements?
PK: Matter is made up of atoms that contain a dense nucleus consisting of positively charged protons and neutrons with negatively charged electrons, equal in number to the protons in the nucleus. Electrons occupy orbitals with discrete energy levels. The numbers of protons and neutrons are unique to the element and determine its position in the periodic table. The highest energy level electrons are valence band electrons; they determine how atoms of an element bond with each other and with other elements. Mechanical concepts including force, energy and work are also assumed known
C01: Electric charge or **charge** for the purpose of circuit analysis is a population of <u>electrons</u> (defined as negative) or of <u>protons</u> (defined as positive), or, in a structure such as a battery, positive or negative ions
C02: Conservation of charge is a physical principle that states that in any process involving <u>charges</u>, no net charge is created or destroyed.
C03: Solid matter has <u>atoms</u> in a fixed arrangement (lattice); at temperatures of interest, atoms vibrate in the lattice
C04: Free electrons, or **conduction [band] electrons,** are <u>electrons</u> with sufficient energy to become freed from the <u>valence band</u> of the parent atom; where a valence electron has been freed, the parent atom becomes a positive ion with a net charge of one proton
C05: A **conducting material (conductor)** is a [solid] material that has <u>free electrons</u> that can move through the material if energy is imparted; conductor is also applied to a piece of conducting material
C06: A dielectric is a material with negligible concentration of <u>free electrons</u> that therefore does not allow charge to move through it

C07: Electrical force between **charges** depends on the spatial distribution of the charges and their signed magnitudes, like charges repelling and unlike charges attracting, and the medium in the surrounding space

C08: An **electric field** in the vicinity of concentration(s) of charge represents the spatial variation of the magnitude and direction of the electrical force experienced by a unit test charge in the space

C09: [Electrical] **potential energy** is the work that has to be done against the forces between charges to bring about a given arrangement of charges

C10: Voltage is the difference in potential energy between two identified points in a circuit

C11: Current is the rate of flow of charge in a defined cross section

C12: Conventional current is the generally adopted convention that current in a chosen direction is positive if positive charge is moving in that direction or negative charge is moving in the opposite direction

C13: Resistivity is a property of a conducting material that determines the relationship between the resulting current per unit area (current density) and the electric field in the material

C14: Resistance is a property of a device made of conducting material quantifying the propensity for the conductor to resist the flow of current when a voltage is applied or exists across the terminals

C15: Temperature coefficient of resistivity quantifies the way that the resistivity of a conducting material (and hence the resistance of a conductor) changes with temperature, linearly in many cases

C16: A **resistor** is a device made of a conducting material, exhibiting resistance between its two terminals

C17: Ohm's Law is a linear relationship between voltage across and current in a resistance; the proportionality constant is the resistance value

C18: A **circuit element** is a self-contained device or representation which has a characteristic relationship between current through the element and voltage between the terminals

C19: Circuit variables are quantities, including voltage and current that describe the predicted or observed behaviour of the circuit

C20: An **electric circuit** or **electrical network** is a set of interconnected circuit elements whose behaviour is observed via circuit variables

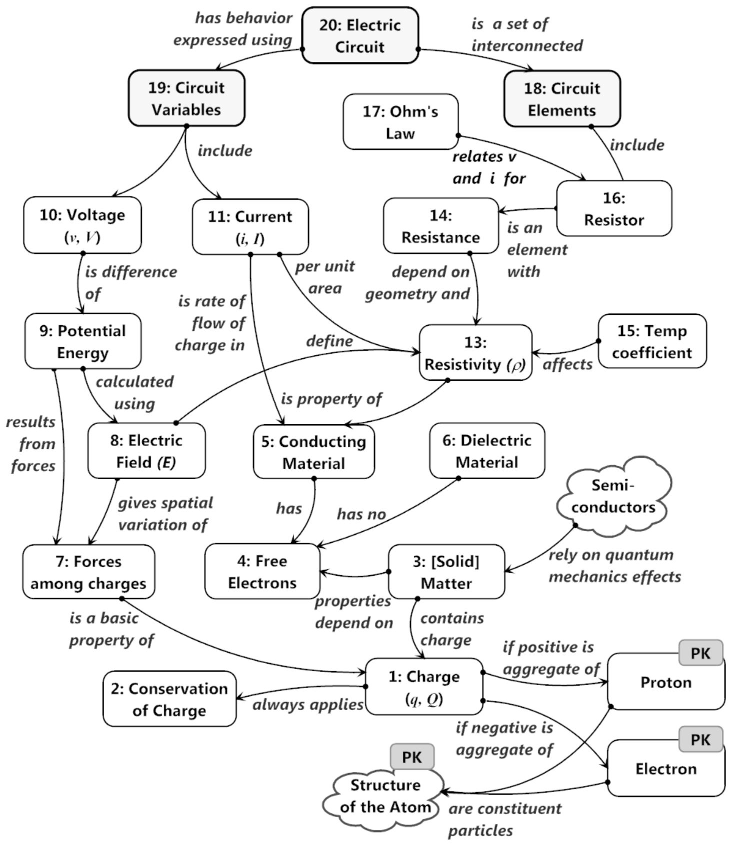


Fig 3: Concept map for the question: Q1: What are the physical origins of circuit variables and elements?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 2: Circuit elements and variables and their behavior

Table III: Concepts for Q2: What are the circuit elements and variables and what is their behavior?

C21: Conservation of energy holds in a self-contained electric circuit: energy may be converted to heat or stored but not created or destroyed.

C22: Energy in an electric circuit is the work done in moving charge (in the form of current) through a potential difference (voltage)

C23: Power in an electric circuit is the rate of change or flow of energy given at any instant by the product of current and voltage

C24: The **power converted to heat** in a resistor with resistance R at any instant is the product of the voltage (v) across its terminals and the current (i) through it or, from Ohms Law, i^2R

C25: A **[sign] convention** for a circuit variable is a method of defining a possibly unknown circuit variable (voltage, current, power, energy), including arbitrarily allocating a positive sign or direction for use in circuit analysis and reconciliation with actual values that may be given or emerge from calculations or measurements

C26: A **waveform** is a description of the variation of a circuit variable with time, usually as an equation or graphically

C27: The **average value** of a waveform is a single figure measure for a circuit variable calculated over a specified period by dividing the integral of the waveform over the period by the period

C28: The **root mean square** value of a waveform is a single figure measure calculated over a specified period by dividing the integral of the squared waveform over the period by the period and taking the square root of the result

C30: An **ideal [independent] voltage source** is a circuit element that maintains a stated voltage (which may vary with time) across its terminals independent of the current at its terminals.

C31: An **ideal [independent] current source** is a circuit element that maintains a stated current (which may vary with time) at its terminals independent of the voltage across its terminals.
C32: An **ideal dependent voltage source** is a circuit element that maintains a voltage across its terminals that is proportional to a control variable, which may be either a voltage or current that may vary with time, independent of the current at its terminals.
C33: An **ideal dependent current source** is a circuit element that maintains a current at its terminals that is proportional to a control variable, which may be either a voltage or current that may vary with time, independent of the voltage across its terminals.
C34: An **active [circuit] element** is a circuit element that is able to supply energy when connected in a circuit
C35: A **passive [circuit] element** is a circuit element that when supplied with energy either converts it to heat or stores it; stored energy may be returned.
C36: An **open circuit** is a hypothetical circuit element that exists between two terminals that allows no flow of current and can be treated in circuit analysis as an infinite resistance between the terminals
C37: A **short circuit** between two terminals is effectively a zero resistance and the current flowing is determined by the remainder of the circuit between the terminals

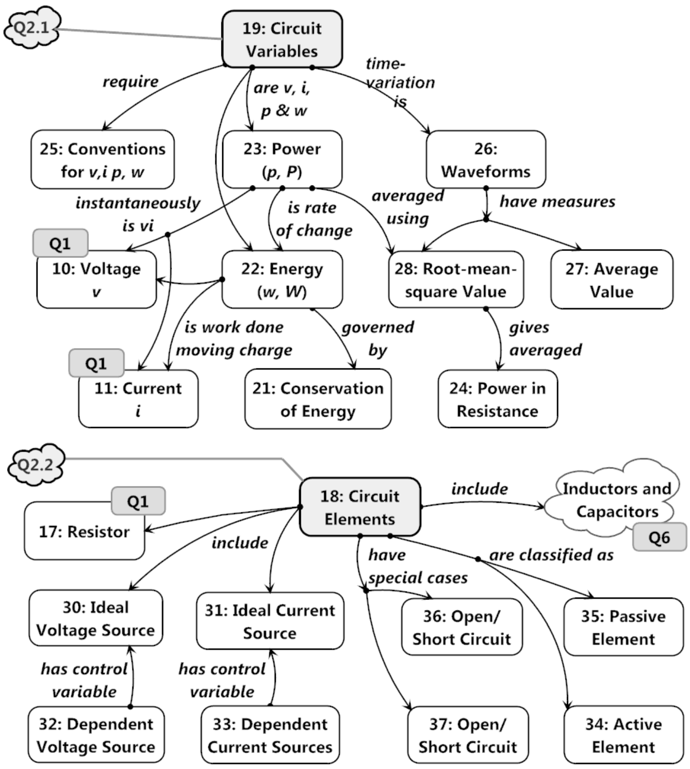


Fig 4: Concept map for the question: Q2: What are the circuit elements and variables and what is their behavior?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 3: Fundamental laws and principles of circuit analysis

Table IV: Concepts for Q3: What are the fundamental laws and principles of circuit analysis?

C40: A **node** is a point in a circuit at which the terminals two or more circuit elements connect; the connecting conductors are assumed to have negligible resistance and the node does not accumulate charge.
C41: A **branch** is formed by a single element between two nodes; the current entering one node always equals the current leaving the other.

C42: A **[closed] loop** in a circuit is path containing a set of circuit elements connected end to end at nodes returning to the chosen starting node and encountering each node only once
C43: A **planar circuit** is one that is possible to draw with no branches crossing
C44: A **mesh** is a region of a planar circuit bounded by branches with no branches crossing the region
C45: The **system view** or **behaviour** of a circuit is the analysis and solution of the circuit that accounts for the collective behaviour of the elements that make up the circuit
C46: Kirchhoff's Voltage Law states that the sum of the voltages defined in a consistent direction around a closed loop in a circuit is zero; this law is based on the principle of conservation of energy
C47: Kirchhoff's Current Law states that at a node in a circuit, the algebraic sum of the currents entering the node is zero; this law is based on the principle of conservation of charge
C48: Solution by simplification: any method, based on Kirchhoff's Laws, of analysing and solving for selected unknown voltages or currents by reducing the complexity of parts of the circuit where each reduced form preserves the system behaviour of the resulting circuit
C49: Systematic solution: any method, based on Kirchhoff's Laws, for analysing and solving for unknown voltages or currents by setting up sufficient simultaneous equations to allow unknown variables in the circuit to be found

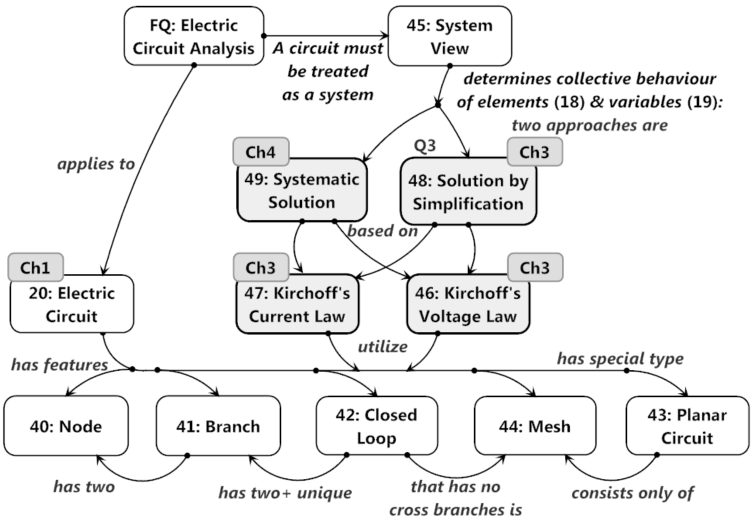


Fig 5 Concept map for the question: Q3: What are the fundamental laws and principles of circuit analysis?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 4: Principles and methods of analysis of a circuit as a system by simplification

Table V: Concepts for Q4: What are the principles and methods of analysis of a circuit as a system by simplifying parts or the whole circuit?

C50: The **equivalent circuit** for a set of interconnected circuit elements with identified terminal nodes is a different set of elements that, if connected at the same set of terminal nodes to an external circuit, would result in the same voltages and currents at those nodes.
C51: Resistor network is a set of resistors connected together with specified nodes that may connect to an external circuit at those nodes.
C52: Circuit elements **connnected in series** are connected end to end at intervening nodes with no other branch connected to any intervening node; all elements have the same value of current
C53: The **equivalent resistance of resistors in series** is a single resistor with resistance equal to the sum of the individual resistances

C54: The **voltage divider rule** relates the voltage across one of two series resistors to the voltage across the combination

C55: Circuit elements are **connnected in parallel** if their respective opposite nodes are connected at two nodes; all elements have the same value of voltage

C56: The **equivalent resistance of resistors in parallel** is a single resistor with resistance with reciprocal equal to the sum of the reciprocals of the individual resistors

C57: The **current divider rule** relates the current in one of two parallel resistors to the terminal current of the combination

C58: Three circuit elements of the same type are **star-connected** if one node of each is connected to form a common node; the remaining three nodes may be connected to an external network

C59: Three circuit elements of the same type are **delta-connected** if connected end to end to form a closed loop; the three nodes at which pairs of elements connect may be connected to an external network

C60: Star-delta transformation allows three resistors connected in one form, say star, to be replaced by the other form, say delta, and conversely, while not affecting the behaviour of an external circuit connected to the three terminals

C61: Redundant circuit elements are those that may be removed from a given circuit without affecting voltages or currents of interest in the remainder of the circuit

C62: Interconnected ideal sources may in some cases be simplified to an equivalent circuit

C63: An intractable circuit is one in which the characteristics of interconnected sources are incompatible and preclude analysis

C65: A **two-terminal network** is a network of arbitrary complexity with all control variables for dependent sources within the network that has two nodes only at which external connections may be made

C66: Thévenin's Theorem states that an arbitrarily complex, two-terminal network has an equivalent circuit consisting of an ideal voltage source in series with a resistor

C67: Norton's Theorem states that an arbitrarily complex, two-terminal network has an equivalent circuit consisting of an ideal current source in parallel with a resistor

C68: Source transformation is based on fact that Thévenin and Norton equivalent circuits are equivalent circuits of each other and one form may be replaced by the other

C69: Open circuit and short circuit tests (or analysis) are methods of finding respectively the Thévenin voltage and Norton current values of a two-terminal network

C70: The **equivalent resistance of an unexcited two-terminal network** determined with all independent source values set to zero (unexcited) is equal to the resistance of the Thévenin equivalent of the network

C71: A **variable load test** is a method of determining Thévenin equivalent circuit parameters; the terminal voltage as the external resistor (load) tends to infinity is the Thévenin voltage and the load resistor that halves the output voltage relative to the open circuit voltage is equal to the Thévenin resistance

C72: The **Maximum Power Transfer Theorem** states that to transfer maximum power from a two-terminal resistive network to a load resistor, the load resistance must be equal to the Thévenin resistance of the network

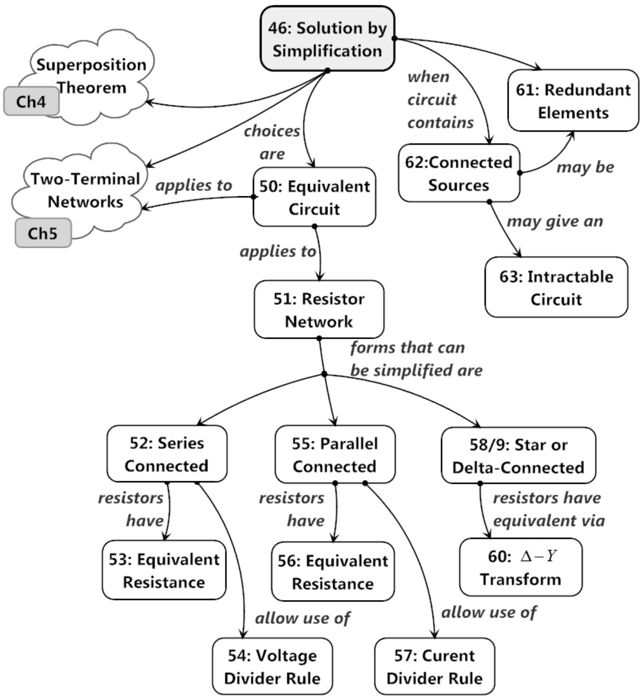


Fig 6: Concept map for the question Q4: What are the principles and methods of analysis of a circuit as a system by simplifying parts or the whole circuit?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 5: Principles and methods of systematic analysis of a circuit

Table VI: Concepts for Q4: What are the principles and methods of analysis of a circuit as a system by systematically setting up equations for solution?

C76: A **reference node** or **datum node** is a node assigned a voltage of zero as reference for measuring the voltages of all other nodes

C77: A **supernode** is an interconnected portion of a network consisting of two or more nodes with independent or dependent voltage source(s) between the nodes

C78: Nodal analysis, or **node-voltage analysis** develops equations for the node voltages relative to the reference node to solve a circuit by applying Kirchhoff 's current law at nodes (other than than nodes in supernodes) and supernodes in the circuit

C79: A **supermesh** is a group of two or more adjacent meshes having common branches containing independent or dependent current sources

C80: Mesh analysis, or **mesh current analysis** develops equations for currents defined around meshes to solve a circuit by applying Kirchhoff 's Voltage Law around meshes excluding those that are part of supermeshes and supermeshes in the circuit

C81: Loop [current] analysis develops equations for sufficient loop currents to solve a circuit (not necessarily planar) by applying Kirchhoff 's Voltage Law around sufficient loops.

C82: A **linear circuit element** is a passive element or a dependent source where stimuli x_1 and x_2 acting alone produce responses y_1 and y_2 respectively, and stimulus $ax_1 + bx_2$ produces response $ay_1 + by_2$ where a and b are constants

C83: A **Linear circuit has** passive elements (R, L, C) and dependent sources each of which are have linear circuit elements

C84: The **Superposition Theorem** states that the unknown voltages or currents in a linear circuit with multiple independent sources may be calculated (using a method of choice) by setting all independent sources except one to zero in turn and summing the individual solutions; dependent sources are not set to zero

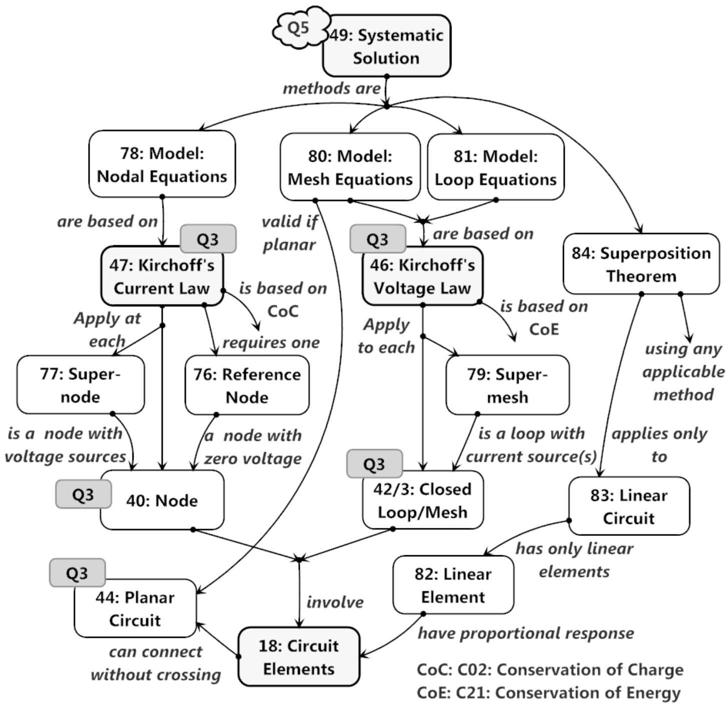


Fig 7: Concept map for the question Q4: What are the principles and methods of analysis of a circuit as a system by systematically setting up equations for solution?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Further Waveform Definitions

Definitions in response to Q2 introduced the concept of a waveform (**C26**). Six specific waveforms that are frequently encountered in electric circuit analysis are defined in Table IX.

Table IX: What are typical waveforms encountered in circuits?

C85: A **step waveform** has value of zero before an initial time and positive value after that time.
C86: A **ramp waveform** has value of zero before an initial time and constant slope staring at zero at the initial time.
C87: A **pulse waveform** is zero before a start time and after an end time and has a defined variation between these two times
C88: A **unit impulse** occurring at a stated time is zero infinitesimally before and after the time of occurrence and has the property that its definite integral is unity
C88: A sinusoidal waveform varies with time according to a sine or cosine function and is specified by three parameters: the amplitude (peak value), frequency (reciprocal of the period) and phase (the offset of the first zero crossing).
C89: An **exponential waveform** has time variation defined by the function exp($-t/\tau$)

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 6: Physical Origins of Capacitors and Inductors

Physical basis and behavior of capacitors

Table X defines concepts related to the capacitor as a circuit element and Fig. 8 maps these concepts. The structure of a capacitor is defined first. The physical processes are defined next: the external charging current creates a separation of charge between the plates resulting in a potential difference. The voltage-current characteristic follows. Properties of series and parallel capacitors as well as the capacitive voltage divider analogous to those for the resistor follow.

Two observed phenomena give rise to concept definitions. Voltage changes across capacitors are relatively slow while current can change instantaneously; hence the voltage flywheel concept. Under certain conditions a capacitor may have voltage across it but have zero current: the capacitor is then described as an open circuit.

Table X: Concept definitions for Question 6(a): What are the physical basis and behavior as circuit elements of (a) capacitors …..

C90: A **capacitor** is a circuit element with structure consisting essentially of two sheets of conducting material, called plates, each connected to a terminal, separated by a sheet of dielectric material, normally of uniform thickness

C91: Charging (and **discharging**) of a capacitance requires a current in an external circuit that adds charge to one plate and removes an equal and opposite charge from the other plate; while no current crosses the dielectric, the current entering one plate is equal and to the current leaving the other plate

C92: Charge stored in a capacitor is equal to the charge deposited on one plate and is given by the integral of the charging current

C93: Electric field associated with a capacitor is due to the equal and opposite charges resident in the metallic plates that collect at the interfaces with the dielectric

C94: Polarisation of the dielectric is the distortion of the atoms in the dielectric as a result of the electric field crossing the dielectric, with electrons bound to atoms tending toward the positive plate and the positively charged atomic nuclei tending toward the negative plate

C95: The **voltage across a capacitor terminals** is proportional to the charge stored and is also equal to the potential difference calculated from the electric field

C96: Capacitance is the constant of proportionality (C) relating the charge stored in a capacitor to the voltage between the plates ($q=Cv$)

C97: The **voltage-current characteristic** for a capacitor gives the current as proportional to the rate of change of voltage, the constant of proportionality being the capacitance, ($i=Cdv/dt$);there is the converse integral form $v= (1/C)\int idt$

C98: Convention for voltage and charging current for a capacitor allows voltage and current directions to be defined arbitrarily; when actual values are obtained, they are interpreted consistently with a current actually positive into a terminal causing that terminal to increase in voltage relative to opposite terminal; a positively charged plate has the more positive potential

C99: The **energy stored in a capacitor** is one half of the voltage squared times the capacitance and with an ideal dielectric can be returned to the external circuit without loss

C100: Capacitor as voltage flywheel captures the fact that a change the voltage across a capacitor requires the transfer of charge and therefore cannot occur instantaneously; the only theoretical exception is if the current is an impulse

C101: Capacitor is effectively an open circuit in a circuit that is in a steady state with constant valued voltage sources; this does not necessarily hold for constant valued current sources

C102: Capacitors in series have an equivalent capacitance whose reciprocal is the sum of the reciprocals of the individual capacitance.

C103: Capacitors in parallel have an equivalent capacitance equal to the sum of the individual capacitances

C104: A **Capacitive Voltage Divider** is made up of two capacitors (capacitance C_1 and C_2) in series and has division ratio C_2 ($C_1 + C_2$) where the output voltage is across C_1

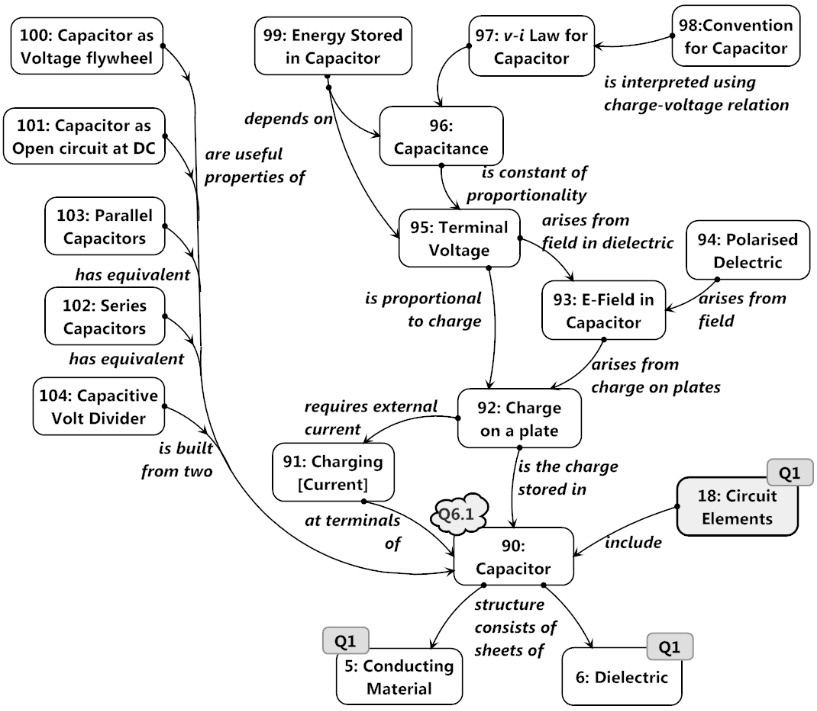


Figure 8. Concept map for Question 6(a): What are the physical basis and behavior as circuit elements of (a) capacitors …

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Physical basis and behavior of inductors

Table XI defines concepts related to the inductor as a circuit element and Fig. 9 maps these concepts. Additional prior knowledge on magnetic phenomena is required as defined in Table XI. The structure of an inductor follows. The physical processes governed by the Laws of Faraday and Lentz are defined next. Inductance is introduced as a single property that represents the physical structure and its linking magnetic field when a current flows. The voltage-current characteristic follows. Properties analogous to those for the series and parallel resistors follow in differential and integral forms. As in the case of the capacitor, the inductor is viewed as a current flywheel and appears to be a short circuit when the current is constant.

Table XI. Concept definitions for Question 6(b): What are the physical basis and behavior as circuit elements of …. (b)inductors?

PK: Magnetic forces occur between any pairs of (i) naturally occurring or man-made permanent magnets, (ii) between current-carrying conductors and (ii) ferromagnetic objects magnetized by a current in a conductor. A **magnetic field** in the vicinity of a magnet or current carrying conductor is a representation of the spatial variation of the magnitude and direction of the magnetic force that would be experienced in the space. **Magnetic flux lines** are imagined lines in the vicinity of a source of magnetic force (permanent magnet or current in a conductor) that depict the direction and strength of the force; the stronger the field the closer the lines. Flux lines close on themselves. **Flux density** is a measure of the force due to a magnetic flux expressed in terms of the flux lines per unit area. **Electromagnetic induction** is the phenomenon of a voltage being produced (induced) between the terminals of a conductor when the flux linking the conductor changes with time

C110: An inductor is a conductor formed into a single- or multi-turn closed shape or coil that in the ideal case has negligible resistance

C111: A **current in an inductor** (the **magnetizing current**) gives rise to magnetic flux lines passing through the turns of the conductor

C112: The **flux linkages** for an inductor is the sum of the flux lines linking each turn

C113: The voltage induced in a conductor when the flux linking the conductor changes is, according to Faraday's Law, proportional to the time rate of change of the flux linkages

C114: Lentz's Law states that the direction of the voltage induced in a conductor when the flux linkages change is such as would cause a current to flow in a direction that opposes the change of flux

C115: The **inductance** or **self-inductance** of an inductor is the constant of proportionality (depending on the geometry, number of turns and surrounding medium), between the time rate of change of current in the conductor and the induced voltage across the conductor.

C116: The **voltage-current characteristic of an inductor** is $v = L di/dt$ where the conventions for v and i are consistent with Lentz's Law

C117: Convention for voltage and current for an inductor allows voltage and current directions to be defined arbitrarily; when actual values are obtained, they are interpreted consistently with a voltage that is actually positive in the defined direction would give a current increasing relative to the defined direction

C118: Energy is stored in an inductor when a current flows; the energy is equal to one half of the current squared times the inductance and may be returned to an external circuit if the inductor is ideal

C119: An **inductor is a** linear circuit element if the flux linkages are proportional to the current in the inductor

C120: A **real inductor** has a conductor with finite resistance which for analysis is in series with the inductance.

C121: Ideal inductors connected in series have an equivalent inductance equal to the sum of the individual inductances

C122: Ideal inductors connected in parallel have an equivalent inductance with reciprocal equal to the sum of the reciprocals of the individual inductances

C123: Inductor as current flywheel expresses the fact that the current in an inductor cannot change instantaneously; the only theoretical exception is if the voltage is an impulse

C124: An **ideal inductor is effectively a short circuit** if the current it carries is constant with time

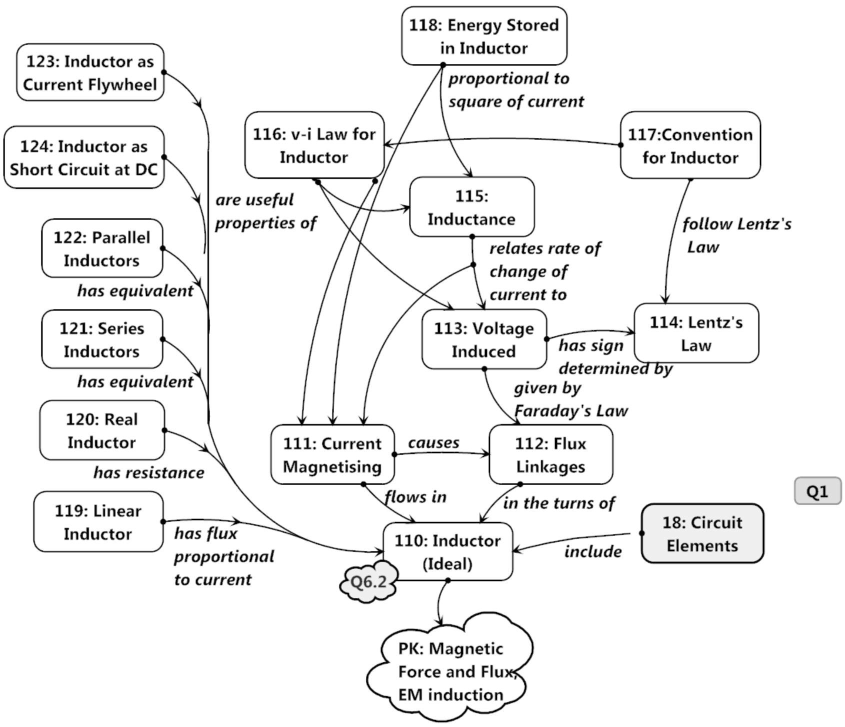


Figure 9. Concept map for Question 6(b): What are the physical basis and behavior as circuit elements of … (b)inductors?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 7: Transients in Circuits

When considering resistive circuits, independent sources may have time-dependent values and Ohms Law would be written $v(t) = i(t)R$. The voltage and current waveforms have the same shape. Responses when multiple sources have different waveforms are the superposition of the individual response waveforms. With capacitors and inductors present, their voltage-current relationships $i_C = Cdv_C/dt$ and $v_L = Ldi_L/dt$ (or their integral forms) in general give rise to different voltage and current waveforms. When resistors, inductors and capacitors are connected in a circuit with independent and possibly dependent sources, analysis and solution of the circuit must satisfy the principle of the circuit as a system (**C45**). The use of Kirchhoff's Voltage and Current Laws (**C46, C47**) in formulating circuit equations ensures compliance. The resulting equations are linear constant coefficient differential equations.

The context of interest is when a circuit with capacitors or inductors operating in an initial state (**C130**), experiences a disturbance (**C131**) in the form of a changed excitation or element. The circuit will move to a new steady state (**C130**). Such changes are described as transients (**C132**) and describing how voltages and currents change as the circuit moves to a new steady state.

Table XII defines a set of concepts for transient analysis of circuits and Fig. 10 presents a map of these concepts.

Table XII: Concept definitions for Question 7: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors with time-varying sources or changes in circuit parameters or elements?

PK: First and second order constant coefficient differential equations: solution method and related terms: forcing function, complete response as complementary function and particular integral.

C130: A **steady state** in a circuit is the condition when all voltages and currents have waveforms that are constant or vary periodically

C131: A **disturbance** in a circuit is a change such as: change to waveform of source(s), switching of source(s), a change in element parameter values or addition or removal of elements from the circuit

C132: A **transient** in a circuit that includes inductors or capacitors is the time-varying process that exists due a disturbance

C133: The state of a circuit with inductors or capacitors is the set of values of inductor current(s) and capacitor voltage(s)

C134: The **initial state** of a circuit is the state at the start of a transient

C135: The **final steady state** of a circuit is the state at the end of a transient, that is, when a steady state is reached

C136: Kirchoff's Voltage and Current Laws are applied in circuits with elements including inductors or capacitors using their respective voltage-current characteristic resulting in one or more constant coefficient linear differential equation

C137: A **first order circuit** has one (or its equivalent) energy storage element and is governed by a first order differential equation

C:138: A **second order circuit** has two energy storage elements (or their equivalents) and is governed by a second order differential equation

C139: The **natural response** of a circuit is the solution to the differential equation with the forcing function set to zero, corresponding to the mathematical term complementary function

C140: The **forced response** of a circuit is the steady state response when the natural response has gone to zero, corresponding to the mathematical term particular integral

C141: The **complete response** of a circuit is the sum of the natural and forced responses

C142: The **time invariance principle** states that, given the solution to a differential equation $x(t)$ when the forcing function is $f(t)$, the solution with forcing function $f(t-T)$ is $x(t-T)$

C143: The zero-input response of a circuit is the solution when the circuit has initial energy storage but no source(s) providing a forcing function

C 144: The **zero-state response** of a circuit is the solution when source(s) provide a forcing function but the initial energy storage is zero

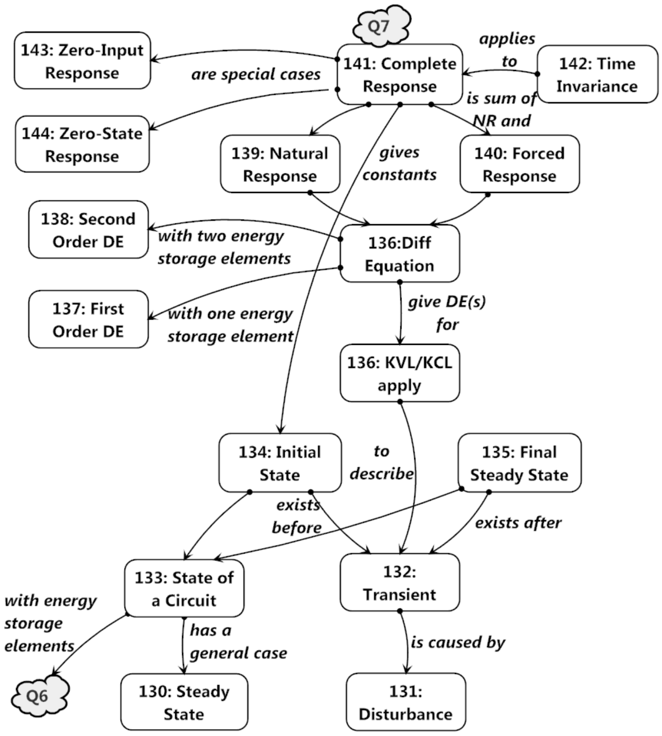


Figure 10: Concept map for Question 7: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors with time-varying sources?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Question 8: Steady State Sinusoidal Analysis

The concept of a circuit in a steady state is introduced in **C130.** In the context of transient analysis. An important for of steady state is the steady state sinusoidal condition defined in **C150**. The set of concepts in Table XII extend the various circuit laws, theorems and rules established in response to Questions 3 to 5 to circuits containing resistors, inductors and capacitors in the sinusoidal steady state.

Two critical concepts are the phasor (**C151**) and impedance (**C155**). The phasor represents sinusoidal voltages and currents by complex numbers and dispenses with the need to use the frequency – which is constant - in equations for voltage and current. Concept **C151** allows values of the waveform to be calculated at any time from the waveform should this prove necessary. As in other cases, voltage and current conventions are essential (**C152**).

The second critical step is the description of individual passive circuit elements as impedances that embody their voltage-current characteristics under steady state sinusoidal conditions (at a fixed frequency). Networks of such elements, incorporating dependent sources as well, have equivalent impedances (**C157**) with the various forms analogous to the cases in resistive circuits: series, parallel and star and delta forms.

Extensions of Thevenin and Norton's Theorems extend to describe two terminal networks with steady state sinusoidal sources and impedances. The notions of source transformation follow as does the Maximum Power Transfer Theorem for the case of complex Thevenin impedance and load impedance.

Table XIII defines a set of concepts for steady state sinusoidal analysis of circuits and Fig. 11 presents a map of these concepts.

Table XIII: Concept definitions for Question 8: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors under sinusoidal steady state conditions?

PK: Complex numbers, rectangular and polar co-ordinates, arithmetic with complex numbers, Euler notation

C150: **Steady state sinusoidal conditions** exist in a circuit if all independent sources have sinusoidal waveforms at a common frequency and element voltages and currents are sinusoidal, the circuit having been excited for sufficiently long for any natural response to have gone to zero

C151: The **phasor representation** for a sinusoidal waveform having a known and constant frequency is a complex number with magnitude equal to the amplitude of the waveform and angle equal to the phase of the waveform; phasors obey the laws of arithmetic for complex numbers

C152: The **sign convention associated with a phasor** voltage or current may be assigned arbitrarily; the voltage-current relationship for an element is written with the same sign on both sides of the equation if the current is shown entering the plus-marked terminal otherwise negative sign must be introduced on one side of the equation

C153: Capacitive reactance is the ratio of the magnitude of the phasor voltage across a capacitor to magnitudes of the phasor current

C154: Inductive reactance is the ratio of the magnitude of the phasor voltage across a capacitor to the magnitude of the phasor current

C155: The **Impedance** or **complex impedance** of a single or network of passive elements with two terminals is the ratio of the complex phasor voltage across the terminals to the complex phasor current passing between the terminals

C156: Kirchhoff's Voltage and Current Laws may be used to apply the methods of systematic and simplification analysis using phasor representations for voltages and currents and impedances in a circuit under steady state sinusoidal conditions

C157: The equivalent impedance of a network of passive circuit elements and dependent sources with control variables completely within the network is a single impedance that may replace the network without affecting the behavior of an external circuit to which it is connected; the real and imaginary parts are the resistive and reactive components respectively

C158: Complex impedances in series have a single equivalent impedance equal to the complex sum of the individual impedances

C159: Complex impedances in parallel have a single equivalent impedance with reciprocal equal to the complex sum of the reciprocals of the individual impedances

C160: Star-Delta transformations may be applied to three impedances connected in one form to obtain the equivalent form.

C161: The **admittance** of an element or equivalent circuit element is the reciprocal of its impedance, with real part called conductance and imaginary part called susceptance

C162: The **Thevenin equivalent** of a two-terminal network under steady state sinusoidal conditions is a voltage source with value expressed as a phasor in series with an impedance

C163: The **Norton equivalent** of a two-terminal network under steady state sinusoidal conditions is a current source with value expressed as a phasor in parallel with an impedance

C164: Source transformations are valid under steady state sinusoidal conditions

C165: The **voltage divider law** for two complex impedances is analogous to that for resistors.

C166: The **current divider law** for two complex impedances is analogous to that for resistors.

C167: The **instantaneous value** of a voltage or current may be recovered from the phasor representation at time t by projecting the tip of the phasor located at angle $2\pi ft+\theta$ onto the horizontal axis for a cosine waveform and onto the vertical axis for the sine waveform with frequency f

C168: The **maximum power transfer theorem under steady state sinusoidal conditions** states that maximum power is delivered by a source to a load impedance when the load impedance is equal to the complex conjugate of the Thevenin impedance of the source

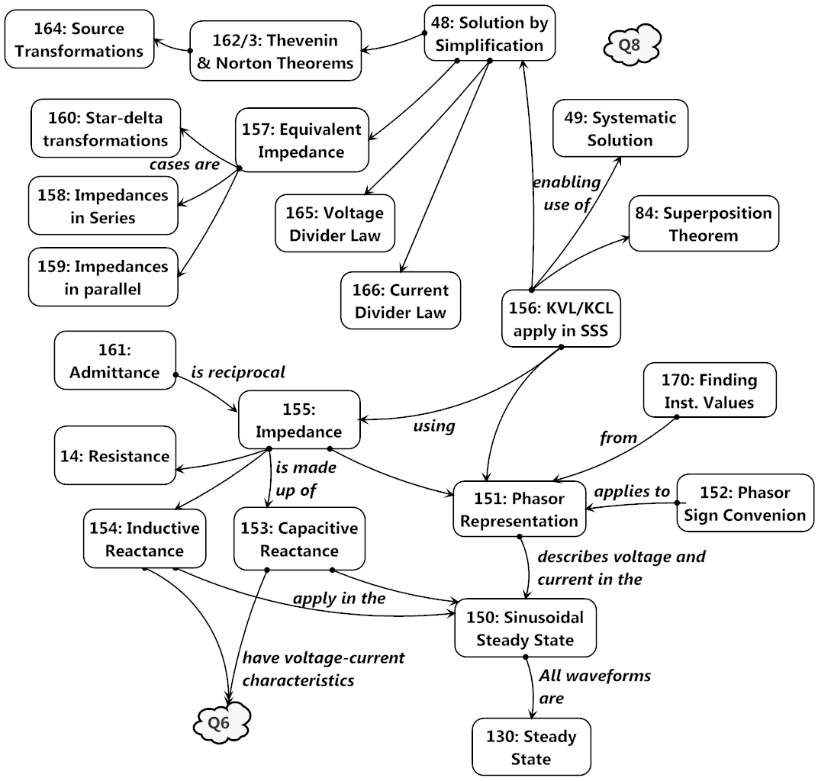


Figure 11: Concept map for Question 8: What is the behavior and analysis method for circuits containing resistors, capacitors and inductors under sinusoidal steady state conditions?

Top	Q1	Q2	Q3	Q4	Q5	Q6-C	Q6-L	Q7	Q8

Estimate of Size of Circuits Knowledge Base

Concept definitions and concept maps for Q6 to Q8 follow. Table VIII contains a measure of the size of the circuits knowledge base.

TABLE VIII
NUMBERS OF CONCEPTS AND LINKS

Question	Table	Figure	Numbers of: Concepts	Links
Q1	II	3	20	28
Q2	III	4	16	18
Q3	IV	5	10	19
Q4	V	6	21	25
Q5	VI	7	10	21
Subtotal			77	111
Q2-addition	IX	-	6	-
Q6…(a)	X	8	15	20
Q6…(b)	XI	9	15	21
Q7	XII	10	15	18
Q8	XIII	11	19	26
Total			148	196