Taylor series expansion

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

how to evaluate the exponential?

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Given an 8 bit (working register, single precision) how many terms do you use?

x has a range $\longrightarrow$ depends on format of the register.

1. Format is unsigned to maximise range.
2. Select format based on real world constraints.

$x > 1$; $\max(e^{x}) \lesssim 30. \longrightarrow \max(x) = 3.4$

3. Select Q 4.3 unsigned.

Max = $11111.111_{2u} \doteq 31.875_{10}$.

Does $\frac{x^n}{n!}$ tend to zero?

i.e. is $\frac{x^{(n+1)}}{(n+1)!} < \frac{x^n}{n!}$

4. Solve for n in $2^{-m} = \frac{x^n}{n!}$ with $\max(x)$.

$n \in \mathbb{N}_0^+$

$n_{\mathbb{R}} = 5.3851$

choose $n = 6$ $(6 > 5.3851)$

* solve for n appropriately.

5. Evaluate 6 terms of factorial.

Implementation

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$

Find inter-term factor

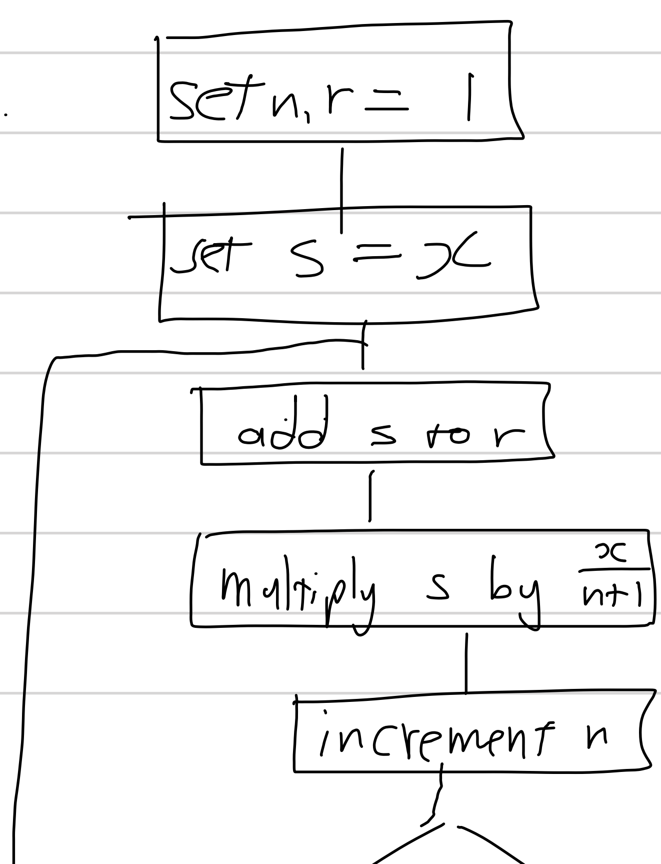
$$\Delta = \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^{n}}{n!}} = \frac{x^{n+1-n}\, n!}{(n+1)!} = \frac{x}{n+1}$$

$$t_n = \frac{x^{n}}{n!} \quad \text{for } x = 3.4$$

$n = 8 : t_8 = 0.443$
$n = 9 : t_9 = 0.167$
$n = 10 : t_{10} = 0.055 \quad (\checkmark < 2^{-m})$

$\Gamma^{-1}(x) \approx$ anti factorial.

$n \equiv$ counter
$r \equiv$ running total
$s \equiv$ inter-term factor

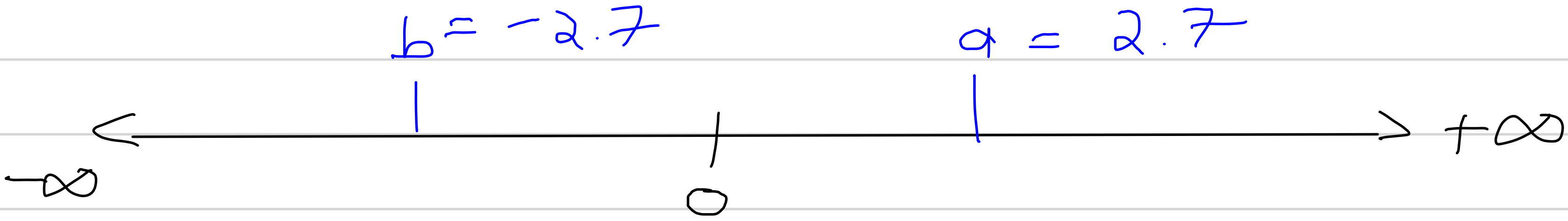


yes

$n < 10$

end

Round, floor, ceil, trunc:



	a	b	Description
Round	3	-3	Round to closest $\mathbb{Z}$ away from 0
Floor	2	-3	Round towards $-\infty$
Ceil	3	-2	Round towards $+\infty$
Trunc	2	-2	Round towards 0