

Number format

In Q3.0 add 101 and 111.

$$\begin{array}{r} 1101 \\ +111 \\ \hline 1100 \end{array} \quad \text{Q3.0}$$

$$\begin{array}{r} 1101 \quad \text{Q3.0} \\ +111 \quad \text{Q3.0} \\ \hline \boxed{0}1100 \quad \text{Q4?} \end{array} \quad \text{Q3 with carry}$$

$$110.110$$

highest order (n) = 2 \Rightarrow Q n.m
lowest order (-m) = -3 \Rightarrow Q 2.3

Binary subtraction

$$x = 1101 - 0101 \Rightarrow$$

$$\begin{array}{r} 1101 \\ -0101 \\ \hline 1000 \end{array} \quad \begin{array}{l} 13 \\ 5 \\ 8 \end{array} \quad \text{Q3}$$

$$x = 1011 - 1100 \Rightarrow$$

$$\begin{array}{r} 1011 \quad 11 \\ -1100 \quad 12 \\ \hline \boxed{0}1111 \quad 31 \end{array} \quad \text{? (wtf?)}$$

(carry)

$$x = 011.0110 - 01.01100$$

$$\begin{array}{r} 011.01100 \\ -001.01100 \\ \hline 010.00000 \end{array} \quad \begin{array}{l} 3.4375_{10} \\ 1.4375_{10} \\ \text{Q2.6} \end{array} \quad 2.0000 \checkmark$$

Signed numbers

$$y = \underbrace{(-1)^{a_{n+1}}}_{\text{sign bit}} \times \underbrace{\sum_{i=-m}^n a_i b^i}_{\text{standard expression}} \quad \left(\begin{array}{l} \text{signed bit form.} \\ \text{or sign and magnitude.} \end{array} \right)$$

$$y = \left\{ \begin{array}{l} \sum_{i=-m}^n a_i b^i : a_n = 0 \\ -\sum_{i=-m}^n \overline{a_i} b^i : a_n = 1 \end{array} \right.$$

$$y = -a_n \times b^n + \sum_{i=-m}^n a_i b^i \quad (\text{two's complement}).$$

↙ signed.

	sign bit	ones comp	two's comp	unsigned
000	+0	+0	0	0
001	1	1	1	1
010	2	2	2	2
011	3	3	3	3
100	-0	-3	-4	4
101	-1	-2	-3	5
110	-2	-1	-2	6
111	-3	-0	-1	7
Range	6	6	7	7

= Max-Min

Range refers to the length of the number line.

$$x = \frac{1}{+0}$$

$$y = \frac{1}{-0}$$

$x=y?$ No!

Signed addition and subtraction (two's complement space)

$$x = 101.01 + 01.10 \quad Q4.2s$$

$$= 11101.01 + 00001.10$$

$$\begin{array}{r} 11101.01 \\ + 00001.10 \\ \hline 11110.11 \end{array}$$

$$-1_{10} \text{ in } Q0s : 1$$

$$-1_{10} \text{ in } Q1s : 11 \quad (-2 + 1 = -1)$$

$$-1_{10} \text{ in } Q4s : 11111 \quad (-16 + 8 + 4 + 2 + 1 = -1)$$

To change format, pad with MSB (above radix).

$$\begin{aligned} 11101.01 \quad Q4.2s \text{ in decimal} &= -16 + 8 + 4 + 1 + 0.25 \\ &= -2.75 \end{aligned}$$

$$00001.10 \quad Q4.2s \text{ in decimal} = 1 + 0.5 = 1.5$$

$$\begin{aligned} 11110.11 \quad Q4.2 \text{ in decimal} &= -16 + 8 + 4 + 2 + 0.5 + 0.25 \\ &= -1.25 \end{aligned}$$