

Unsigned: $x = \sum_{j=-m}^n a_j b^j$

S&M: $x = -1^{a_n} \times \sum_{j=-m}^n a_j b^j$

Two's: $x = -a_n b^n + \sum_{j=-m}^{n-1} a_j b^j$

Ones $x = \begin{cases} \sum_{j=-m}^{n-1} a_j b^j & ; a_n = 0 \\ -\sum_{j=-m}^{n-1} a_j b^j & ; a_n = 1 \end{cases}$

Binary	Unsigned ₁₀	S&M ₁₀	Two's ₁₀	Ones ₁₀
1111	15	-7	-1	-0
1110	14	-6	-2	-1
1101	13	-5	-3	-2
1100	12	-4	-4	-3
1011	11	-3	-5	-4
1010	10	-2	-6	-5
1001	9	-1	-7	-6
1000	8	0	-8	-7
0111	7	+7	+7	+7
0110	6	+6	+6	+6
0101	5	+5	+5	+5
0100	4	+4	+4	+4
0011	3	+3	+3	+3
0010	2	+2	+2	+2
0001	1	+1	+1	+1
0000	0	+0	+0	+0

Carry: Answer length greater than problem space.
Overflow: Sign bit changes incorrectly.

Example

$$\begin{array}{r} '11'0'11'.110 \\ + 11100.111 \\ \hline [1] 11000.101 \end{array}$$

signed
signed
Carry, no overflow.

$$\begin{array}{r} '01101 \\ + 01001 \\ \hline 10110 \end{array}$$

signed
signed
Overflow, no carry.

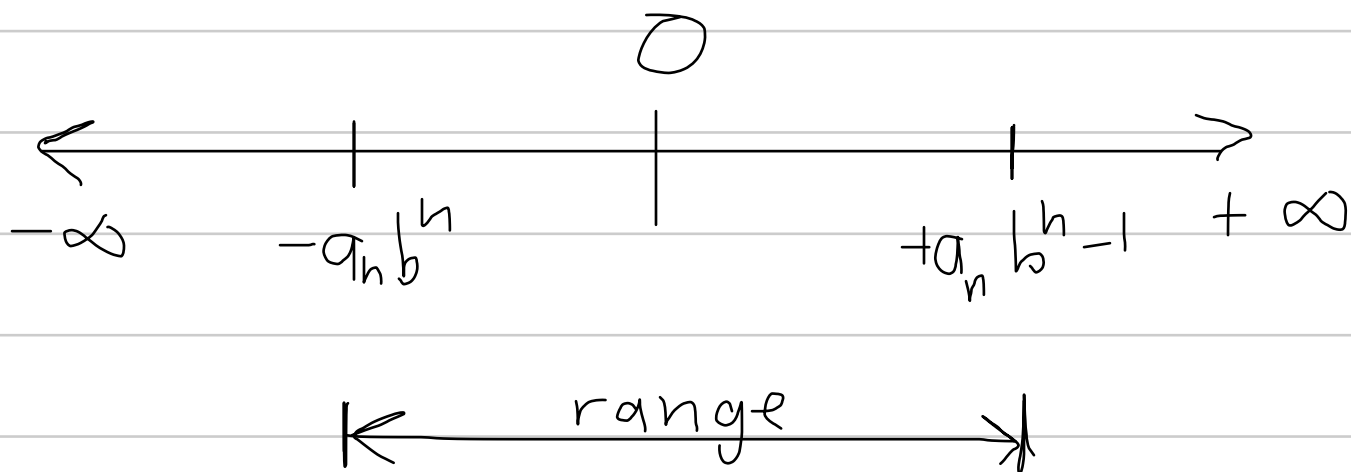
$$\begin{array}{r} '0'1101 \\ + 11101 \\ \hline 101010 \end{array}$$

signed
signed
carry, no overflow

$$\begin{array}{r} 10101 \\ + 10101 \\ \hline [1] 01010 \end{array}$$

signed
signed
carry and overflow

1.7 Signed range and Resolution



$$\begin{aligned} \text{Range} &= a_n b^n - 1 - (-a_n b^n) & a_n &= 1 \\ &= 2a_n b^n - 1 \end{aligned}$$

$$\text{If } b = 2$$

$$\text{Range} = a_n b^{n+1} - 1$$

$$\text{Resolution} = b^{-m}$$

1.8 Detecting overflow

Self study exercise ... to be discussed next week

1.9 Multiplication

AND

$$1 \times 1 = 1$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$0 \times 0 = 0$$

XOR

$$1 \oplus 1 = 0$$

$$1 \oplus 0 = 1$$

$$0 \oplus 1 = 1$$

$$0 \oplus 0 = 0$$

OR

$$1 + 1 = 1$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$0 + 0 = 0$$

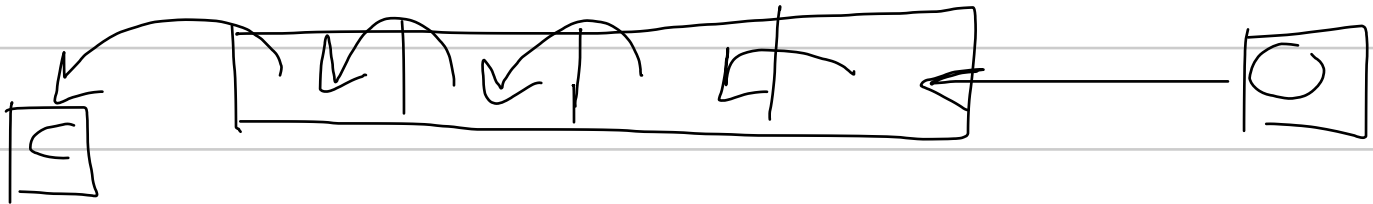
NOT

$$\overline{1} = 0$$

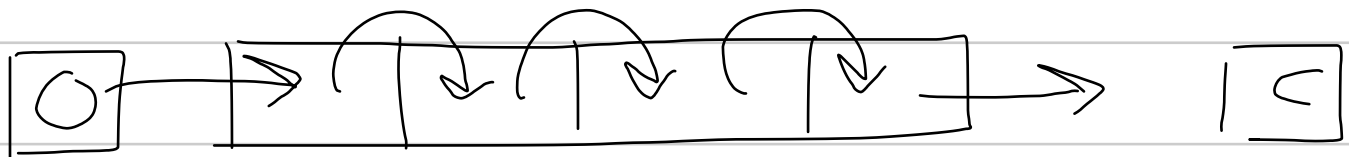
$$\overline{0} = 1$$

Shifting (Logical)

Left:



Right



Arithmetic shifts

$$\begin{array}{l} 1111 = -1_{10} \text{ signed} \\ 1111111111111111 = -1_{10} \text{ signed} \end{array}$$

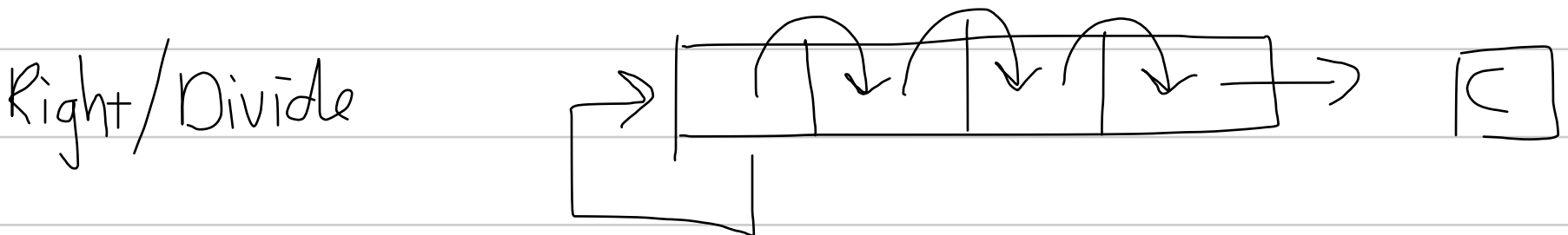
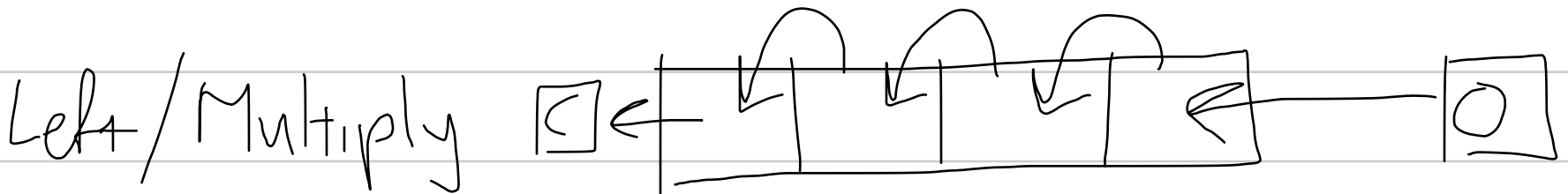
$$1110 = -2$$

$-2 = 1110$ signed
 $-1 = 1111$ signed
 $+2 = 0010$ signed
 $+1 = 0001$ signed

$-5 = 1011$

$+6 = 0110$!! Overflow

Arithmetic shift



Example

```

      011
    x 010
    -----
      000
     0110
    -----
    00000
    -----
    00110
  
```

Unsigned

$$\begin{array}{r} 110011 \\ \times 010101 \\ \hline 110011 \end{array}$$

Q5.0

Q5.0



00000000

11001100

00000000

11001100

00000000

$$1000010111 \quad Q10.0$$