University of the Witwatersrand, Johannesburg
School of Electrical and Information Engineering
ELEN7015 Teletraffic Engineering

Tutorial Sheet 4: Queueing Theory: Beyond M/M/n Queues

Exercise 1: For the mathematically inclined: Prove Palm's identity:

$$m_i = \int_0^\infty X^i p_x(X)\,dx = \int_0^\infty iX^{i-1}[1 - P_x(X)]\,dx$$

What are the specific forms for the mean and variance?

Exercise 2: Consider the 1024:128 concentrator shown in figure 3.1. Assuming the offered traffic per line is 0.09 Erl, determine the blocking probability if the concentrator is implemented as a single switch.

For the three-stage implementation in figure 3.1, estimate the blocking probability with the same level of offered traffic.

Exercise 3: For the 1024:128 line concentrator, the proposal is to use 64:16 switches in the first stage, 16:8 in the second stage and 8:8 in the third stage. Draw the switch, showing the interconnections between stages. (Show only the first, second and last modules in each stage.)

Recalculate the the blocking probability.

Exercise 4: A 1000-line to 120 channel concentrator is proposed. The 120 output channels are implemented as four 30-channel TDM links. Compare two implementations: a) any of the thousand input channels can access any available output time slot; and b) the input lines are allocated to four 250-line groups and each group can access only one TDM line.

Exercise 5: A queue with Markov arrivals with $\lambda = 1$ has a single server which has a deterministic service time with holding time $H = 0.8$. Determine the average queue length and delay by the most appropriate method.

Exercise 6: The paper by Gans *et al* proposes a method for estimating the mean waiting time for a small call centre as

$$E[\text{Wait for M/G/N}] \simeq E[\text{Wait for M/M/N}] \times \frac{1 + C^2}{2}$$

where C is the coefficient of variation of the service time.

Apply this method to a queue with Markov arrivals with $\lambda = 1$, a single server which has a general service time distibution with holding time $H = 0.6$ and standard deviation $\simeq= 0.25$. Compare the result to that given by the Pollaczek-Khintchine equations.
