

Tutorial Sheet 3: Queueing Theory and M/M/n Queues

Exercise 1: What do the following mean:

M/G/2/10, G/M/4, M/Ek/1/ - FCFS

Exercise 2: A queueing system operates on M/M/1 basis. Tabulate the state probabilities for different values of $\rho = 0.1, 0.3, 0.7$ and 0.9 . State your criterion for cutting off the series. What difference does it make using the finite or infinite state formulas?

Repeat for an M/M/2 system.

Exercise 3: Develop the equations for the state probabilities and the constant p_0 for a M/M/3 queue with arrival rate λ and three servers each capable of a service rate of μ . First draw the Markov chain diagram.

Exercise 4: Equation 2.36 indicates that the effect of truncating a state probability series for an M/M/n queue after the Q th state falls off with ρ^{Q+1} . Test this assertion for M/M/1 and M/M/N queues for $Q = 10$ and 20 and the following values of $\rho = 0.1, 0.2, 0.5, 0.8$

Exercise 5: A stream of packets passes through four routers which can be assumed to have Poissonian service times. What is the mean and standard deviation of the delay across the entire route. What delay will not be exceeded 90% of the time?

Exercise 6:

A queue operates with a utilisation of 0.6. The mean service time is $10 \mu\text{s}$ and the standard deviation is $2.3 \mu\text{s}$. What delay can an arriving packet expect on the average for the following numbers of packets in the queue: 0, 1, 2, 4, 8?

Exercise 7: A model for a service process is as follows. The observed relative frequencies of service times is constant up to one minute. For longer service times, the density falls linearly to three minutes. No service takes longer than 4 min. Find the average waiting time if the utilisation is 40%.

Exercise 8: A queueing system has a finite population, M , of parties that can request service. The service request rate per party is λ' . The service process is carried out by a single server with rate μ . Draw the Markov Chain diagram for this process.

Exercise 9: A finite single server queueing system is required to operate at $\rho = 0.7$. Determine the queue sizes to achieve blocking probability of 1%, 5% and 10%.
