Digital Transmission through the Additive White Gaussian Noise Channel

ELEN 3024 - Communication Fundamentals

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Digital Transmission Through the AWGN Channel

Proakis and Salehi, "Communication Systems Engineering" (2nd Ed.), Chapter 7

Overview

Introduction

Convert output of a signal source into a sequence of binary digits

Now consider transmission of digital information sequence over communication channels characterized as additive white Gaussian noise channels

AWGN channel $\rightarrow$ one of the simplest mathematical models for various physical communication channels

Most channels are analog channels $\rightarrow$ digital information sequence mapped into analog signal waveforms

Introduction

Focus on:

- characterization, and
- design

of analog signal waveforms that carry digital information and performance on an AWGN channels

Consider both baseband and passband signals.

Baseband $\rightarrow$ no need for carrier

passband channel $\rightarrow$ information-bearing signal impressed on a sinusoidal carrier

7.4.Multidimensional Signal Waveforms

Previous section $\rightarrow$ signal waveforms in two dimensions

Consider design of a set of $M = 2^k$ signal waveforms having more than two dimensions

First, consider M mutually orthogonal signal waveforms (each waveform has dimension $N = M$)

7.4.1. Orthogonal Signal Waveforms - baseband

Fig. 7.24. $\rightarrow$ 2 sets of $M = 4$ orthogonal signal waveforms

set of K baseband signal waveforms $\rightarrow$ Gram-Schmidt $\rightarrow$ $M \leq K$ mutually orthogonal signal waveforms

M signal waveforms are simply the orthonormal signal waveforms $\psi_i, i = 1, 2, \ldots, M$ obtained from Gram-Schmidt procedure

7.4.1. Orthogonal Signal Waveforms - baseband

When M orthogonal signal waveforms are nonoverlapping in time $\rightarrow$ digital information conveyed by time interval (PPM)

$$s_m(t) = Ag_T(t-(m-1)T/M), \quad \begin{array}{l} m = 1, 2, , \ldots, M \\ (m-1)T/M \leq t \leq mT/M \end{array}$$

$g_T(t)$ signal pulse of duration T/M

Practical reasons $\rightarrow$ all M signal waveforms have same energy

7.4.1. Orthogonal Signal Waveforms - baseband

Example $\rightarrow$ M PPM signals, all signals have amplitude A:

$$
\begin{aligned}
\int_0^T s_m^2(t)dt &= \int_{(m-1)T/M}^{mT/M} g_T^2(t-(m-1)T/M)dt \\
&= A^2 \int_0^{T/M} g_T^2(t)dt \\
&= \mathcal{E}_s, \text{ all } m
\end{aligned}
$$

7.4.1. Orthogonal Signal Waveforms - baseband

Geometric representation for PPM $\rightarrow$ M basis functions:

$$\psi_m(t) = \begin{cases} \frac{1}{\sqrt{\mathcal{E}}} g(t-(m-1)T/M), & (m-1)T/M \leq t \leq mT/M \\ 0, & \text{otherwise} \end{cases}$$

M-ary PPM signal waveforms are represented geometrically by the M-dimensional vectors:

$$\begin{aligned} \mathbf{s_1} &= (\sqrt{\mathcal{E}_s}, 0, 0, \ldots, 0) \\ \mathbf{s_2} &= (0, \sqrt{\mathcal{E}_s}, 0, \ldots, 0) \\ &\vdots \\ \mathbf{s_M} &= (0, 0, 0, \ldots, \sqrt{\mathcal{E}_s}) \end{aligned}$$

7.4.1. Orthogonal Signal Waveforms - baseband

$\mathbf{s_i}$ and $\mathbf{s_j}$ orthogonal $\rightarrow \mathbf{s_i} \cdot \mathbf{s_j} = 0$

M signal vectors are mutually equidistant, i.e.,

$$d_{mn} = \sqrt{||\mathbf{s_m} - \mathbf{s_n}||^2} = \sqrt{2\mathcal{E}_s}, \forall\ m \neq n$$

7.4.1. Orthogonal Signal Waveforms - bandpass Signals

Bandpass orthogonal signals $\rightarrow$ set of baseband orthogonal waveforms $s_m(t), m = 1, 2, \ldots, M$ multiplied with carrier $\cos 2\pi f_c t$

Thus:

$$u_m(t) = s_m(t) \cos(2\pi f_c t), \quad \begin{array}{l} m = 1, 2, \ldots, M \\ 0 \leq t \leq T \end{array}$$

Energy in each of the bandpass signal waveforms is one-half of the energy of the corresponding baseband signal waveforms

7.4.1. Orthogonal Signal Waveforms - bandpass Signals

Orthogonality:

$$\begin{aligned}\int_0^T u_m(t)u_n(t) &= \int_0^T s_m(t)s_n(t)\cos^2 2\pi f_c t dt \\ &= \frac{1}{2}\int_0^T s_m(t)s_n(t)dt + \frac{1}{2}\int_0^T s_m(t)s_n(t)cos 4\pi f_c t dt \\ &= 0\end{aligned}$$

$f_c \gg$ bandwidth baseband signals

7.4.1. Orthogonal Signal Waveforms - bandpass Signals

M-ary PPM signals achieve orthogonality in time domain by means of nonoverlapping pulses

Alternative $\rightarrow$ construct a set of M carrier-modulated signals which achieve orthogonality in frequency domain $\rightarrow$ carrier-frequency modulation

Simplest form $\rightarrow$ frequency-shift keying

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying

Simplest form of frequency modulation $\rightarrow$ binary frequency-shift keying

Use f_1 and $f_2 = f_1 + \Delta f$ to convey binary data

$$u_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos 2\pi f_1 t, \; 0 \leq t \leq T_b$$

$$u_2(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos 2\pi f_2 t, \; 0 \leq t \leq T_b$$

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying

M-ary FSK $\rightarrow$ transmit a block of $k = \log_2 M$ bits/signal waveform

$$u_m(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos(2\pi f_c t + 2\pi m \Delta f t), \; m = 0, 1, \ldots, M-1$$

M frequency waveforms have equal energy $\mathcal{E}_s$

Frequency separation Δf determines the degree to which we can discriminate among the M possible signals.

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying

Measure of similarity → correlation coefficients γ_{mn}

$$\gamma_{mn} = \frac{1}{\mathcal{E}_s} \int_0^T u_m(t) u_n(t) dt$$

Substitution:

$$\begin{aligned}
\gamma_{mn} &= \frac{1}{\mathcal{E}_s} \int_0^T \frac{2\mathcal{E}_s}{T} \cos(2\pi f_c t + 2\pi m \Delta f t) \cos(2\pi f_c t + 2\pi n \Delta f t) dt \\
&= \frac{1}{T} \int_0^T \cos 2\pi (m-n) \Delta f t dt \\
&\quad + \frac{1}{T} \int_0^T \cos[4\pi f_c t + 2\pi (m+n) \Delta f t] dt \\
&= \frac{\sin 2\pi (m-n) \Delta f T}{2\pi (m-n) \Delta f T}
\end{aligned}$$

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying

Refer to Fig. 7.26

Signal waveforms are orthogonal when Δf is a multiple of $\frac{1}{2T}$

Minimum value of the correlation coefficient is $\gamma_{mn} = -0.217$, for $\Delta f = 0.715$

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying

M-ary orthogonal FSK waveforms have a geometric representation as M, M-dimensional orthogonal vectors, given as:

$$\begin{aligned} \mathbf{s_1} &= (\sqrt{\mathcal{E}_s}, 0, 0, \ldots, 0) \\ \mathbf{s_2} &= (0, \sqrt{\mathcal{E}_s}, 0, \ldots, 0) \\ &\vdots \\ \mathbf{s_M} &= (0, 0, 0, \ldots, \sqrt{\mathcal{E}_s}) \end{aligned}$$

with basis functions $\psi_m(t) = \sqrt{\frac{2}{T}} \cos 2\pi(f_c + m\Delta f)t$

Distance between pair of signal vectors is $d = \sqrt{2\mathcal{E}_s}$ for all m, n (minimum distance)

7.4.1. Orthogonal Signal Waveforms - Frequency-Shift Keying