Digital Transmission through the Additive White Gaussian Noise Channel

ELEN 3024 - Communication Fundamentals

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Digital Transmission Through the AWGN Channel

Proakis and Salehi, “Communication Systems Engineering” (2nd Ed.), Chapter 7

Overview

Introduction

Convert output of a signal source into a sequence of binary digits

Now consider transmission of digital information sequence over communication channels characterized as additive white Gaussian noise channels

AWGN channel $\rightarrow$ one of the simplest mathematical models for various physical communication channels

Most channels are analog channels $\rightarrow$ digital information sequence mapped into analog signal waveforms

Introduction

Focus on:

- characterization, and
- design

of analog signal waveforms that carry digital information and performance on an AWGN channels

Consider both baseband and passband signals.

Baseband $\rightarrow$ no need for carrier

passband channel $\rightarrow$ information-bearing signal impressed on a sinusoidal carrier

7.1.Geometric Representation of Signal Waveforms

Gram-Schmidt orthogonalization $\rightarrow$ construct an orthonormal basis for a set of signals

Develop a geometric representation of signal waveforms as points in a signal space

Representation provides a compact characterization of signal sets, simplifies analysis of performance

Using vector representation, waveform communication channels are represented by vector channels (reduce complexity of analysis)

7.1.Geometric Representation of Signal Waveforms

Suppose set of M signal waveforms $s_m(t)$, $1 \leq m \leq M$ to be used for transmitting information over comms channel

From set of M waveforms, construct set of $N \leq M$ orthonormal waveforms $\rightarrow$ N dimension of signal space

Use Gram-Schmidt orthogonalization procedure

7.1.1. Gram-Schmidt Orthogonalization Procedure

Given first waveform $s_1(t)$, with energy $\mathcal{E}_1 \rightarrow$ first waveform of the orthonormal set:

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{\mathcal{E}_1}}$$

7.1.1. Gram-Schmidt Orthogonalization Procedure

Second waveform $\rightarrow$ constructed from $s_2(t)$ by computing the projection of $s_2(t)$ onto $\psi_1(t)$:

$$c_{21} = \int_{-\infty}^{\infty} s_2(t)\psi_1(t)dt$$

Then, $c_{21}\psi_1(t)$ is subtracted from $s_2(t)$ to yield:

$$d_2(t) = s_2(t) - c_{21}\psi_1(t)$$

7.1.1. Gram-Schmidt Orthogonalization Procedure

$d_2(t)$ is orthogonal to ψ_1, but energy of $d_2(t) \neq 1$.

$$\psi_2(t) = \frac{d_2(t)}{\sqrt{\mathcal{E}_2}}$$

$$\mathcal{E}_2 = \int_{-\infty}^{\infty} d_2^2(t) dt$$

7.1.1. Gram-Schmidt Orthogonalization Procedure

In general, the orthogonalization of the kth function leads to

$$\psi_k(t) = \frac{d_k(t)}{\sqrt{\mathcal{E}_k}}$$

where

$$d_k t = s_k(t) - \sum_{i=1}^{k-1} c_{ki}\psi_i(t)$$

$$\mathcal{E}_k = \int_{-\infty}^{\infty} d_k^2(t)dt$$

and

$$c_{ki} = \int_{-\infty}^{\infty} s_k(t)\psi_i(t)dt, \; i = 1, 2, \ldots, k-1$$

7.1.1. Gram-Schmidt Orthogonalization Procedure

Orthogonalization process is continued until all the M signal waveforms $\{s_m(t)\}$ have been exhausted and $N \leq M$ orthonormal waveforms have been constructed

The N orthonormal waveforms $\{\psi_n(t)\}$ forms a basis in the N-dimensional signal space.

Dimensionality $N = M$ if all signal waveforms are linearly independent.

7.1.1. Gram-Schmidt Orthogonalization Procedure

Example 7.1.1

Selfstudy

7.1.1. Gram-Schmidt Orthogonalization Procedure

Can express the M signals $\{s_m(t)\}$ as exact linear combinations of the $\{\psi_n(t)\}$

$$s_m(t) = \sum_{n=1}^{N} s_{mn}\psi_n(t), \ m = 1, 2, \ldots, M$$

where

$$s_{mn} = \int_{-\infty}^{\infty} s_m(t)\psi_n(t)dt$$

$$\mathcal{E}_m = \int_{-\infty}^{\infty} s_m^2(t)dt = \sum_{n=1}^{N} s_{mn}^2$$

Thus

$$\mathbf{s_m} = (s_{m1}, s_{m2}, \ldots, s_{mN})$$

7.1.1. Gram-Schmidt Orthogonalization Procedure

Energy of the mth signal $\rightarrow$ square of length of vector or square of Euclidean distance from origin to point in N-dimensional space.

Inner product of two signals equal to inner product of their vector representations

$$\int_{-\infty}^{\infty} s_m(t) s_n(t) dt = \mathbf{s_m} \cdot \mathbf{s_n}$$

Thus, any N-dimensional signal can be represented geometrically as a point in the signal space spanned by the N orthonormal functions $\{\psi_n(t)\}$

7.1.1. Gram-Schmidt Orthogonalization Procedure

Example 7.1.2

Selfstudy

7.1.1. Gram-Schmidt Orthogonalization Procedure

Set of basis functions $\{\psi_n(t)\}$ obtained by Gram-Schmidt procedure is not unique

7.2. Pulse Amplitude Modulation

Pulse Amplitude Modulation $\rightarrow$ information conveyed by the amplitude of the transmitted signal

7.2.1. Baseband Signals

Binary PAM $\rightarrow$ simplest digital modulation method

Binary 1 $\rightarrow$ pulse with amplitude A

Binary 0 $\rightarrow$ pulse with amplitude $-A$

Also referred to as binary antipodal signalling

Pulses transmitted at a bit rate $R_b = 1/T_b$ bits/sec ($T_b \rightarrow$ bit interval)

7.2.1. Baseband Signals

Generalization of PAM to nonbinary (M-ary) pulse transmission straightforward

Instead of transmitting one bit at a time, binary information sequence is subdivided into blocks of k bits $\rightarrow$ symbol

Each symbol represented by one of $M = 2^k$ pulse amplitude values

$k = 2 \rightarrow M = 4$ pulse amplitude values

When bitrate R_b is fixed, symbol interval

$$T = \frac{k}{R_b} = kT_b$$

7.2.1. Baseband Signals

In general M-ary PAM signal waveforms may be expressed as

$$s_m(t) = A_m g_T(t), \quad m = 1, 2, \dots, M, \quad 0 \le t \le T$$

where $g_T(t)$ is a pulse of some arbitrary shape (example $\rightarrow$ Fig. 7.7.)

Distinguishing feature among the M signals is the signal amplitude

All the M signals have the same pulse shape

7.2.1. Baseband Signals

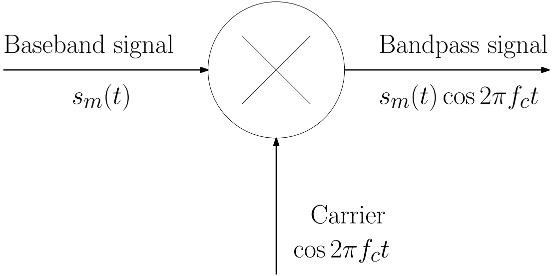
Another important feature → energies

$$
\begin{aligned}
\mathcal{E}_m &= \int_0^T s_m^2(t)dt \\
&= A_m^2 \int_0^T g_T^2(t)dt \\
&= A_m^2 \mathcal{E}_g, \qquad\qquad m = 1, 2, \ldots, M
\end{aligned}
$$

$\mathcal{E}_g$ is the energy of the signal pulse $g_T(t)$

7.2.2. Bandpass Signals

To transmit digital waveforms through a bandpass channel by amplitude modulation, the baseband signal waveforms $s_m(t),\ m = 1, 2, \ldots, M$ are multiplied by a sinusoidal carrier of the form $\cos 2\pi f_c t$



7.2.2. Bandpass Signals

Transmitted signal waveforms:

$$u_m(t) = A_m g_T(t) \cos 2\pi f_c t, \quad m = 1, 2, \ldots, M$$

Amplitude modulation $\rightarrow$ shifts the spectrum of the baseband signal by an amount f_c $\rightarrow$ places signal into passband of the channel

Fourier transform of carrier: $\left[\delta(f - f_c) + \delta(f + f_c)\right]/2$

7.2.2. Bandpass Signals

Spectrum of amplitude-modulated signal

$$U_m(t) = \frac{A_m}{2}\left[G_T(f - f_c) + G_T(f + f_c)\right]$$

Spectrum of baseband signal $s_m(t) = A_m g_T(t)$ is shifted in frequency by amount f_c

Result $\rightarrow$ DSB-SC AM $\rightarrow$ Fig. 7.9

Upper sideband $\rightarrow$ frequency content of $u_m(t)$ for $f_c < |f| \leq f_c + W$

Lower sideband $\rightarrow$ frequency content of $u_m(t)$ for $f_c - W \leq |f| < f_c$

$u_m(t) \rightarrow$ bandwidth $= 2W \rightarrow$ twice bandwidth of baseband signal

7.2.2. Bandpass Signals

Energy of bandpass signal waveforms $u_m(t),\ m = 1, 2, \ldots, M$

$$
\begin{aligned}
\mathcal{E}_m &= \int_{-\infty}^{\infty} u_m^2(t) dt \\
&= \int_{-\infty}^{\infty} A_m^2 g_T^2(t) \cos^2 2\pi f_c t \ dt \\
&= \frac{A_m^2}{2} \int_{-\infty}^{\infty} g_T^2(t) \ dt + \frac{A_m^2}{2} \int_{-\infty}^{\infty} g_T^2(t) \cos 4\pi f_c t \ dt
\end{aligned}
$$

When $f_c \gg W$

$$\int_{-\infty}^{\infty} g_T^2(t) \cos 4\pi f_c t \ dt = 0$$

Thus,

$$\mathcal{E}_m = \frac{A_m^2}{2} \int_{-\infty}^{\infty} g_T^2(t) = \frac{A_m^2}{2} \mathcal{E}_g$$

7.2.2. Bandpass Signals

$\mathcal{E}_g \rightarrow$ energy in the signal $g_T(t)$

Energy in bandpass signal is one-half of the energy of the baseband signal

Assume $g_T(t)$

$$g_T(T) = \begin{cases} \sqrt{\frac{\mathcal{E}_g}{T}} & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow$ amplitude-shift keyeing (ASK)

7.2.3. Geometric Representation of PAM Signals

Baseband signals for M-ary PAM $\rightarrow s_m(t) = a_m g_T(t),\ M = 2^k$, $g_T(t)$ pulse with peak amplitude normalized to unity

M-ary PAM waveforms are one-dimensional signals, expressed as

$$s_m(t) = s_m \psi(t),\ m = 1, 2, \ldots, M$$

basis function $\psi(t)$

$$\psi(t) = \frac{1}{\sqrt{\mathcal{E}_g}} g_T(t),\ 0 \leq t \leq T$$

$\mathcal{E}_g \rightarrow$ energy of signal pulse $g_T(t)$

7.2.3. Geometric Representation of PAM Signals

signal coefficients $\rightarrow$ one-dimensional vectors

$$s_m = \sqrt{\mathcal{E}_g} A_m, \quad m = 1, 2, \ldots, M$$

Important parameter $\rightarrow$ Euclidean distance between two signal points:

$$d_{mn} = \sqrt{|s_m - s_n|^2} = \sqrt{\mathcal{E}_g (A_m - A_n)^2}$$

$\{A_m\}$ symmetrically spaced about zero and equally distant between adjacent signal amplitudes $\rightarrow$ symmetric PAM

Refer to Fig 7.11

7.2.3. Geometric Representation of PAM Signals

PAM signals have different energies.

Energy of mth signal

$$\mathcal{E}_m = s_m^2 = \mathcal{E}_g A_m^2, \quad m = 1, 2, \ldots, M$$

Equally probable signals, average energy is given as:

$$\mathcal{E}_{av} = \frac{1}{M} \sum_{m=1}^{M} \mathcal{E}_m = \frac{\mathcal{E}_g}{M} \sum_{m=1}^{M} A_m^2$$

7.2.3. Geometric Representation of PAM Signals

If signal amplitudes are symmetric about origin

$$A_m = (2m - 1 - M), \quad m = 1, 2, \ldots, M$$

Average energy

$$\mathcal{E}_{av} = \frac{\mathcal{E}_g}{M} \sum_{m=1}^{M} (2m - 1 - M)^2 = \mathcal{E}_g (M^2 - 1)/3$$

7.2.3. Geometric Representation of PAM Signals

When baseband PAM impressed on a carrier, basic geometric representation of the digital PAM signal waveforms remain the same

Bandpass signal waveforms $u_m(t)$ expressed as

$$u_m(t) = s_m \psi(t)$$

where

$$\psi(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \cos 2\pi f_c t$$

and

$$s_m = \sqrt{\frac{\mathcal{E}_g}{2}} A_m, \quad m = 1, 2, \ldots, M$$

7.3. Two-Dimensional Signal Waveforms

PAM signal waveforms are basically one-dimensional signals

Now consider the construction of two-dimensional signals

7.3.1 Baseband Signals

Two signal waveforms $s_1(t)$ and $s_2(t)$ orthogonal over interval $(0, T)$ if

$$\int_0^T s_1(t)s_2(t)dt = 0$$

Fig. 7.12 $\rightarrow$ two examples

$$\begin{aligned}\mathcal{E} &= \int_0^T s_1^2(t)dt = \int_0^T s_2^2(t)dt = \int_0^T [s_1']^2(t)dt = \int_0^T [s_2']^2(t)dt \\ &= A^2 T\end{aligned}$$

Either pair of these signals may be used to transmit binary information, one signal waveform $\rightarrow$ 1, the other waveform $\rightarrow$ 0

7.3.1 Baseband Signals

Geometrically, signal waveforms represented as signal vectors in two-dimensional space

One choice, select unit energy, rectangular functions

$$\psi_1(t) = \begin{cases} \sqrt{2/T}, & 0 \le t \le T/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\psi_2(t) = \begin{cases} \sqrt{2/T}, & T/2 < t \le T \\ 0, & \text{otherwise} \end{cases}$$

7.3.1 Baseband Signals

Signal waveforms $s_1(t)$ and $s_2(t)$ expressed as

$$\begin{aligned} s_1(t) &= s_{11}\psi_1(t) + s_{12}\psi_2(t) \\ s_2(t) &= s_{21}\psi_2(t) + s_{22}\psi_2 t \end{aligned}$$

where

$$\mathbf{s_1} = (s_{11}, s_{12}) = (A\sqrt{T/2}, A\sqrt{T/2})$$
$$\mathbf{s_2} = (s_{21}, s_{22}) = (A\sqrt{T/2}, -A\sqrt{T/2})$$

Fig 7.13 $\rightarrow$ plot of $\mathbf{s_1}$ and $\mathbf{s_2}$

Signals are separated by 90^o $\rightarrow$ orthogonal

7.3.1 Baseband Signals

Square of length of each vector gives the energy in each signal

$$\mathcal{E}_1 = ||\mathbf{s_1}||^2 = A^2 T$$
$$\mathcal{E}_2 = ||\mathbf{s_2}||^2 = A^2 T$$

Euclidean distance between two signals is

$$d_{12} = \sqrt{||\mathbf{s_1} - \mathbf{s_2}||^2} = A\sqrt{2T} = \sqrt{2A^2 T} = \sqrt{2\mathcal{E}}$$

$\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E} \rightarrow$ signal energy

Similarly:

$$\mathbf{s_1}' = (A\sqrt{T}, 0) = (\sqrt{\mathcal{E}}, 0)$$
$$\mathbf{s_2}' = (0, A\sqrt{T}) = (0, \sqrt{\mathcal{E}})$$

Euclidean distance between $\mathbf{s_1}'$ and $\mathbf{s_2}'$ identical to that of $\mathbf{s_1}$ and $\mathbf{s_2}$

7.3.1 Baseband Signals

Suppose we wish to construct four signal waveforms in two dimensions

Four signal waveforms $\rightarrow$ transmit 2 bits in signalling interval of length T

use $-\mathbf{s_1}$ and $-\mathbf{s_2}$

Obtain 4-point signal constellation $\rightarrow$ Fig. 7.15

$s_1(t)$ and $s_2(t)$ orthogonal, plus $-s_1(t)$ and $-s_2(t)$ orthogonal $\rightarrow$ biorthogonal signals

7.3.1 Baseband Signals

Procedure for constructing a larger set of signal waveforms relatively straightforward

add additional signal points (signal vectors) in two-dimensional plane, construct corresponding waveforms by using the two orthonormal basis functions $\psi_1(t)$ and $\psi_2(t)$

Suppose construct $M = 8$ two-dimensional signal waveforms, all of equal energy $\mathcal{E}$.

Fig. 7.16 $\rightarrow$ constellation diagram

Transmit 3 bits at a time

7.3.1 Baseband Signals

Remove condition that all 8 waveforms have equal energy

Example: select 4 biorthogonal waveforms with energy $\mathcal{E}_1$ and another 4 biorthogonal waveforms with energy $\mathcal{E}_2$ ($\mathcal{E}_2 > \mathcal{E}_1$)

Refer to Fig. 7.17

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Bandpass PAM $\rightarrow$ set of baseband signals impressed on carrier

Similarly, set of M two-dimensional signal waveforms $s_m(t),\ m = 1, 2, \ldots, M$ create a set of bandpass signal waveforms

$$u_m(t) = s_m(t) \cos 2\pi f_c t, \quad m = 1, 2, \ldots, M, \quad 0 \leq t \leq T$$

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Consider special case in which M two-dimensional bandpass signal waveforms constrained to have same energy:

$$
\begin{aligned}
\mathcal{E}_m &= \int_0^T u_m^2(t)dt \\
&= \int_0^T s_m^2(t)\cos^2 2\pi f_c t dt \\
&= \frac{1}{2}\int_0^T s_m^2(t)dt + \frac{1}{2}\int_0^T s_m^2(t)\cos 4\pi f_c t dt \\
&= \frac{1}{2}\int_0^T s_m^2(t)dt \\
&= \mathcal{E}_s, \text{ for all } m
\end{aligned}
$$

When all signal waveforms have same energy, corresponding signal points fall on circle with radius $\sqrt{\mathcal{E}_s}$

Fig. 7.15 $\rightarrow$ example of constellation with $M = 4$

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Signal points equivalent to a single signal whose phase is shifted $\rightarrow$ carrier-phase modulated signal

$$u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \quad M = 0, 1, \ldots, M-1,$$

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

When $g_T(t)$ rectangular pulse

$$g_T(t) = \sqrt{\frac{2\mathcal{E}_s}{T}}, \ \ 0 \leq t \leq T$$

Corresponding transmitted signal waveforms

$$u_m(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right),$$

has constant envelope, carrier phase changes abruptly at beginning of each signal interval

$\Rightarrow$ phase-shift keyeing (PSK)

Fig 7.18. QPSK signal waveform

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Can rewrite carrier-phase modulated signal equation as

$$u_m(t) = g_T(t)A_{mc}\cos 2\pi f_c t - g_T(t)A_{ms}\sin 2\pi f_c t$$

where

$$\begin{aligned} A_{mc} &= \cos 2\pi m/M \\ A_{ms} &= \sin 2\pi m/M \end{aligned}$$

Phase-modulated signal may be viewed as two quadrature carriers with amplitudes $g_T(t)A_{mc}$ and $g_T(t)A_{ms}$ (Fig. 7.19)

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Thus, digital phase-modulated signals can be represented geometrically as two-dimensional vectors

$$\mathbf{s_m} = (\sqrt{\mathcal{E}_s} \cos 2\pi m/M, \sqrt{\mathcal{E}_s} \sin 2\pi m/M)$$

Orthogonal basis functions are

$$\psi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \cos 2\pi f_c t$$

$$\psi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \sin 2\pi f_c t$$

Fig. 7.20 $\rightarrow$ signal point constellations for M = 2,4,8

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Mapping or assignment of k information bits into the $M = 2^k$ possible changes may be done in number of ways

Preferred mapping $\rightarrow$ Gray encoding (Fig. 7.20)

Most likely errors caused by noise $\rightarrow$ selection of an adjacent phase to transmitted phase $\rightarrow$ single bit error

7.3.2 Two-dimensional Bandpass Signals - Carrier-Phase Modulation

Euclidean distance between any two signal points in constellation

$$
\begin{aligned}
d_{mn} &= \sqrt{||\mathbf{s_m} - \mathbf{s_n}||^2} \\
&= \sqrt{2\mathcal{E}_s\left(1 - \cos\frac{2\pi(m-n)}{M}\right)}
\end{aligned}
$$

Minimum Euclidean distance (distance between two adjacent signal points)

$$d_{min} = \sqrt{2\mathcal{E}_s\left(1 - \cos\frac{2\pi}{M}\right)}$$

$d_{min} \rightarrow$ determine error-rate performance of receiver in AWGN

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

When $\mathcal{E}_s$ not equal for every symbol, we can impress separate information "bits" on each of the quadrature carriers ($\cos 2\pi f_c t$ and $\sin 2\pi f_c t$) $\rightarrow$ Quadrature Amplitude Modulation (QAM)

Form of quadrature-carrier multiplexing

$$u_m(t) = A_{mc} g_T(t) \cos 2\pi f_c t + A_{ms} g_T(t) sin 2\pi f_c t, \quad m = 1, 2, \ldots, M$$

$\{A_{mc}\}$ and $\{A_{ms}\}$ are the sets of amplitude levels obtained by mapping k-bit sequences into signal amplitudes.

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

Fig. 7.21 $\rightarrow$ 16-QAM $\rightarrow$ amplitude modulating each quadrature carrier by $M = 4$ PAM

QAM $\rightarrow$ combined digital-amplitude and digital-phase modulation

$$u_{mn}(t) = A_m g_T(t)\cos(2\pi f_c t + \theta_n), \quad \begin{array}{l} m = 1, 2, \ldots, M_1, \\ n = 1, 2, \ldots, M_2 \end{array}$$

If $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$ $\rightarrow$

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

Fig. 7.21 → 16-QAM → amplitude modulating each quadrature carrier by $M = 4$ PAM

QAM → combined digital-amplitude and digital-phase modulation

$$u_{mn}(t) = A_m g_T(t) \cos(2\pi f_c t + \theta_n), \quad \begin{array}{l} m = 1, 2, \ldots, M_1, \\ n = 1, 2, \ldots, M_2 \end{array}$$

If $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$ → $k_1 + k_2 = \log_2(M_1 \times M_2)$ bits, at symbol rate $R_b/(k_1 + k_2)$

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

Fig. 7.22. $\rightarrow$ Functional block diagram of modulator for QAM

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

Geometric signal representation of the signals:

$$\mathbf{s_m} = (\sqrt{\mathcal{E}_s} A_{mc}, \sqrt{\mathcal{E}_s} A_{ms})$$

Fig. 7.23 $\rightarrow$ Examples of signal space constellations for QAM.

Average transmitted energy $\rightarrow$ sum of the average energies on each of the quadrature carriers

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

For rectangular signal constellations, average energy/symbol

$$\mathcal{E}_{av} = \frac{1}{M} \sum_{i=1}^{M} ||\mathbf{s}_i||^2$$

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation

Euclidean distance

$$d_{mn} = \sqrt{||\mathbf{s_m} - \mathbf{s_n}||^2}$$

7.3.3 Two-dimensional Bandpass Signals - Quadrature Amplitude Modulation