Angle Modulation

ELEN 3024 - Communication Fundamentals

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Angle Modulation

Proakis and Salehi, “Communication Systems Engineering” (2nd Ed.), Chapter 3

Overview

3.3.3 Implementation Angle Modulators and Demodulators

Modulation process $\rightarrow$ generation of new frequencies not present in input signal

Modulation + Demodulation $\neq$ linear, time-invariant

Angle modulators $\rightarrow$ time-varying, nonlinear systems

3.3.3 Implementation Angle Modulators and Demodulators

One way of generating FM $\rightarrow$ design oscillator whose frequency changes with input voltage

Input voltage 0 $\rightarrow$ oscillator generates sinusoid with frequency f_c

When input voltage changes, frequency changes

Two approaches to design VCO (voltage-controlled oscillator)

- Varactor diode
- Reactance tube

3.3.3 Implementation Angle Modulators and Demodulators

Varactor diode $\rightarrow$ capacitor whose capacitance changes with applied voltage

If used in tuned circuit of oscillator, and message signal is applied to it, frequency of oscillator changes according to message

Assume:

- Inductance of inductor $\rightarrow L_0$
- Capacitance of varactor $\rightarrow C(t) = C_0 + k_0 m(t)$

3.3.3 Implementation Angle Modulators and Demodulators

When $m(t) = 0 \rightarrow$ frequency of tuned circuit $f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$

For nonzero $m(t)$

$$
\begin{aligned}
f_i(t) &= \frac{1}{2\pi\sqrt{L_0(C_0 + k_0 m(t)}} \\
&= \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \\
&= f_c \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}}
\end{aligned}
$$

3.3.3 Implementation Angle Modulators and Demodulators

Assuming that:

$$\epsilon = \frac{k_0}{C_0} m(t) \ll 1$$

and using approximations

$$\sqrt{1+\epsilon} \approx 1 + \frac{\epsilon}{2}$$

$$\frac{1}{1+\epsilon} \approx 1 - \epsilon$$

We obtain

$$f_i(t) \approx f_c \left(1 - \frac{k_0}{2C_0} m(t)\right)$$

3.3.3 Implementation Angle Modulators and Demodulators

$$f_i(t) \approx f_c \left(1 - \frac{k_0}{2C_0} m(t) \right)$$

$\Rightarrow$ relation for frequency-modulated signal

3.3.3 Implementation Angle Modulators and Demodulators

Reactance tube, very similar

3.3.3 Implementation Angle Modulators and Demodulators

Indirect method:

First generate a narrowband angle-modulated signal, then change to wideband signal

Fig. 3.32 $\rightarrow$ Generation of narrowband angle-modulated signal

3.3.3 Implementation Angle Modulators and Demodulators

Next step: convert narrowband angle-modulated wave to wideband (Fig. 3.33)

1. frequency multiplier, multiplies instantaneous frequency with constant n

$$u_n(t) = A_c \cos(2\pi f_c t + \phi(t))$$

$$y(t) = A_c \cos(2\pi n f_c t + n\phi(t))$$

3.3.3 Implementation Angle Modulators and Demodulators

2. up- or down-conversion to shift modulated signal to desired carrier frequency

$$u(t) = A_c \cos(2\pi(nf_c - f_{LO})t + n\phi(t))$$

can choose n and can choose f_{LO}

3.3.3 Implementation Angle Modulators and Demodulators

FM Demodulators $\rightarrow$ generate AM signal whose amplitude is proportional to the instantaneous frequency of FM signal, then use AM demodulator to recover message signal

3.3.3 Implementation Angle Modulators and Demodulators

1. Transform FM signal to AM $\rightarrow$ pass FM through LTI system whose frequency response $\approx$ straight line in frequency band of FM signal

If frequency response is:

$$|H(f)| = V_0 + k(f - f_c) \text{ for } |f - f_c| < \frac{B_c}{2}$$

and input to system is

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right),$$

then output

$$v_0(t) = A_c(V_0 + kk_f m(t)) \cos\left(2\pi f_c t + 2\pi k_f \int_{t}^{-\infty} m(\tau)d\tau\right)$$

3.3.3 Implementation Angle Modulators and Demodulators

2. Demodulate signal to obtain $A_c(V_0 + kk_f m(t))$, from which $m(t)$ can be recovered

Many circuits can be used to implement first stage

- Differentiator $|H(f)| = 2\pi f$
- Rising half of a tuned circuit

Balanced modulator $\rightarrow$ two circuits tuned at f_1 and f_2 $\rightarrow$ linear characteristic over wider range of frequencies

FM demodulation where converted to AM $\rightarrow$ bandwidth $= B_c$ (channel bandwidth)

Noise passed by the demodulator $\rightarrow$ noise contained within B_c

3.3.3 Implementation Angle Modulators and Demodulators

FM Demodulation → use feedback in FM demodulator to narrow bandwidth of FM detector, reduce noise power at output of demodulator

Fig 3.37: FM demodulator with feedback

Bandwidth of the discriminator and lowpass filter → match bandwidth of message signal $m(t)$

Output of lowpass filter → desired output

3.3.3 Implementation Angle Modulators and Demodulators

Alternative: Use PLL $\rightarrow$ Fig. 3.38

input:

$$u(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

For FM:

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau$$

3.3.3 Implementation Angle Modulators and Demodulators

VCO $\rightarrow$ generates sinusoid of a fixed frequency, carrier frequency f_c, in absence of input control voltage

Suppose control voltage to VCO is output of loop filter ($v(t)$)

instantaneous frequency of VCO:

$$f_v(t) = f_c + k_v v(t)$$

k_v deviation constant (Hz/Volt)

3.3.3 Implementation Angle Modulators and Demodulators

Thus, VCO output expressed as

$$y_v(t) = A_v \sin[2\pi f_c t + \phi_v(t)]$$

where

$$\phi_v(t) = 2\pi k_v \int_0^t v(\tau) d\tau$$

3.3.3 Implementation Angle Modulators and Demodulators

Phase comparator $\rightarrow$ multiplier and filter that rejects signal component at $2f_c$

Output of phase comparator:

$$e(t) = \frac{1}{2} A_v A_c \sin[\phi(t) - \phi_v(t)]$$

where difference $\phi(t) - \phi_v(t) \equiv \phi_e(t)$ constitutes the phase error

$e(t) \rightarrow$ input to loop filter

Assume PLL in lock $\rightarrow$ phase error small:

$$\sin[\phi(t) - \phi_v(t)] \approx \phi(t) - \phi_v(t) = \phi_e(t)$$

3.3.3 Implementation Angle Modulators and Demodulators

Under this condition (phase error small), deal with linearized model of PLL (Fig. 3.39)

Express phase error as

$$\phi_e(t) = \phi(t) - 2\pi k_v \int_0^t v(\tau) d\tau$$

or

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v v(t) = \frac{d}{dt}\phi(t)$$

or

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v \int_0^\infty \phi_e(\tau) g(t-\tau) d\tau = \frac{d}{dt}\phi(t)$$

3.3.3 Implementation Angle Modulators and Demodulators

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v \int_0^\infty \phi_e(\tau) g(t-\tau) d\tau = \frac{d}{dt}\phi(t)$$

Taking Fourier:

$$(j2\pi f)\mathbf{\Phi_e}(f) + 2\pi k_v \mathbf{\Phi_e}(f) G(f) = (j2\pi f)\mathbf{\Phi}(f)$$

Hence

$$\mathbf{\Phi_e}(f) = \frac{1}{1 + \frac{k_v}{jf} G(f)} \mathbf{\Phi}(f)$$

Corresponding equation for control voltage to the VCO

$$\begin{aligned} V(f) &= \mathbf{\Phi_e}(f) G(f) \\ &= \frac{G(f)}{1 + \frac{k_v}{jf} G(f)} \mathbf{\Phi}(f) \end{aligned}$$

3.3.3 Implementation Angle Modulators and Demodulators

Suppose that we design $G(f)$ such that

$$\left| k_v \frac{G(f)}{jf} \right| \gg 1$$

in frequency band $|f| < W$.

Then:

$$V(f) = \frac{j2\pi f}{2\pi k_v} \mathbf{\Phi_e}(f)$$

In time domain:

$$\begin{aligned} v(t) &= \frac{1}{2\pi k_v} \frac{d}{dt} \phi(t) \\ &= \frac{k_f}{k_v} m(t) \end{aligned}$$

3.3.3 Implementation Angle Modulators and Demodulators

Since control voltage of the VCO is proportional to message signal, $v(t)$ is the demodulated signal

Output of loop filter $(G(f)) \rightarrow$ desired message signal $\therefore$ bandwidth of $G(f) = W$

Noise at the output of loop filter also limited to the bandwidth W.