hrs	/ /20		Exams Office Use Only

University of the Witwatersrand, Johannesburg

Course or topic No(s)	ELEN3015
Course or topic name(s) Paper Number & title	Data and Information Management
Examination/Test* to be held during month(s) of (*delete as applicable)	June 2018
Year of Study (Art & Sciences leave blank)	Third
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	BSc (Eng)(Elec)
Faculty/ies presenting candidates	Engineering
Internal examiners and telephone number(s)	Prof. L. Cheng (x7228)
External examiner(s)	Prof. K. Ouahada
Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)	None

Time allowance

Course Nos	ELEN3015	Hours	3

Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)	Answer *ALL* questions. Closed book Engineering calculator permitted A4 handwritten information sheet Total marks: 105 - Full marks: 100

Internal Examiners or Heads of School are requested to sign the declaration overleaf

1. As the Internal Examiner/Head of School, I certify that this question paper is in final form, as approved by the External Examiner, and is ready for reproduction.

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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.All terms and symbols are as defined in the course handouts. Answers written on your question paper will NOT be marked. Answers written in pencil will NOT be marked.

Question 1

Two parties communicate securely over an open channel using a combined monoalphabetic and columnar transposition cipher scheme. Someone eavesdrops a ciphertext as follows:

ijwmjtgjjwswdrj tntxcyizmuhlhtw wjtwjtxryxxgjhx xjyqnyktjtmtyas

(a) In the first stage, show the frequency analysis method to cryptanalyze the monoalphabetic ciphertext by using the ETAOIN rule.

(5 marks)

(b) In the second stage, show the method to cryptanalyze the columnar transposition ciphertext by using the bigram (assume the anticipated column width is less than 6).

(5 marks)

i. Show the sums of frequency of different possible solutions.

(5 marks)

ii. Show the most likely plaintext.

(5 marks)

(Total 20 marks)

$$\begin{pmatrix} th & 1.52\% & en & 0.55\% & ng & 0.18\% \\ he & 1.28\% & ed & 0.53\% & of & 0.16\% \\ in & 0.94\% & to & 0.52\% & al & 0.09\% \\ er & 0.94\% & it & 0.50\% & de & 0.09\% \\ an & 0.82\% & ou & 0.50\% & se & 0.08\% \\ re & 0.68\% & ea & 0.47\% & le & 0.08\% \\ nd & 0.63\% & hi & 0.46\% & sa & 0.06\% \\ at & 0.59\% & is & 0.46\% & si & 0.05\% \\ on & 0.57\% & or & 0.43\% & ar & 0.04\% \\ nt & 0.56\% & ti & 0.34\% & ve & 0.04\% \\ ha & 0.56\% & as & 0.33\% & ra & 0.04\% \\ es & 0.56\% & te & 0.27\% & ld & 0.02\% \\ st & 0.55\% & et & 0.19\% & ur & 0.02\% \end{pmatrix}$$

Question 2

(a) A memoryless information source has a countably infinite symbol alphabet $\mathbf{S} = \{S_1, S_2, \ldots\}$ with $P_i = b\alpha^i$ for $i = 1, 2, \ldots$. Express b in terms of α.

(5 marks)

(b) Calculate the entropy of $\mathbf{S}$ as a function of α.

(5 marks)

Hint: $\sum_{i=1}^{\infty} a^i = \frac{a}{1-a}$ and $\sum_{i=1}^{\infty} ia^i = \frac{a}{(1-a)^2}$ given $|a| \leq 1$.

(Total 10 marks)

Question 3

The frequency band between 100 kHz and 101 kHz is allocated to a communication system. The signal power is $S = 31$ power unit per hertz. The noise in the band is additive white Gaussian noise with single-sided power spectral density $N_0 = 1$ power unit per hertz.

(a) What is the Shannon limit on the achievable data rate (bits/sec)?

(5 marks)

(b) For a given bandwidth between 800 MHz and 850 MHz, and transmission data rate of 10^5 bits/sec, what is the required signal-to-noise ratio in decibels (dB)?

(5 marks)

(Total 10 marks)

Question 4

Consider a (7, 4) Hamming code with parity-check matrix,

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Add one more parity check bit with the binary-sum value of all the existing 7 bits.

(a) What is the new parity-check matrix?

(2 marks)

(b) What is the minimum Hamming distance of the new code? Prove it.

(4 marks)

(c) If the received sequence is [0 0 1 0 1 E 0 0] (E denotes erasure error), determine the sent codeword using the syndrome decoding algorithm.

(6 marks)

(Total 12 marks)

Question 5

We consider the Galois field GF(2^3) based on the primitive polynomial $h(x) = 1 + x^2 + x^3$.

(a) Derive the Galois field based on the given primitive polynomial in terms of binary sequences, polynomial notations and powers of the primitive element (α).

(5 marks)

(b) Derive the corresponding minimum polynomials.

(5 marks)

(c) Derive the generator polynomial of a single-error-correcting code based on the minimum polynomials. What is the rate of the code generated by the derived generator polynomial?

(5 marks)

(d) If the received sequence is [0 E 1 0 1 E 0] (E denotes erasure error), determine the sent codeword using the syndrome decoding algorithm.

(7 marks)

(Total 22 marks)

Question 6

Consider a systematic binary cyclic code with the generator polynomial $g(x) = x + 1$ (Assume the number of inputs is k).

(a) Determine if the weight of any codeword in this code is even. Give a proof of your argument.

(5 marks)

(b) Determine the minimum Hamming distance of this code. Give a proof of your argument.

(5 marks)

(c) Give an implementation as a convolutional encoder with shift-registers. Draw the connections of the shift-registers.

(5 marks)

(Total 15 marks)

Question 7

For a 3-error-correcting (15, 9) Reed-Solomon code with the generator polynomial

$$\begin{aligned} g(x) =& (x+1)(x+\alpha)(x+\alpha^2)\dots(x+\alpha^5) \\ =& 1+\alpha^4 x+\alpha^2 x^2+\alpha x^3+\alpha^{12}x^4+\alpha^9 x^5+x^6 \end{aligned}$$

over the Galois field defined by Table 2.

Suppose the received word is $10\alpha^2\alpha\alpha^{12}\alpha^9\alpha^8 00000000$, complete the following decoding process by using the Berlekamp-Massey algorithm.

NB: ***Each place marked by the question mark in the following derivations requires an answer. Show all workings.***

The received sequence can be written in the polynomial

$$w(x) = \underline{?\qquad\qquad\qquad\qquad\qquad}.$$

(1 marks)

(a) We get the syndrome polynomials as:

$$
\begin{aligned}
s_0 =& w(\alpha^0) = \underline{?},\\
s_1 =& w(\alpha^1) = \alpha^4,\\
s_2 =& w(\alpha^2) = \alpha^8,\\
s_3 =& w(\alpha^3) = \alpha^{13},\\
s_4 =& w(\alpha^4) = \alpha^7,\\
s_5 =& w(\alpha^5) = \alpha^{11}
\end{aligned}
$$

(1 marks)

(b)

$$
\begin{aligned}
q_{-1}(x) =& \underline{?\qquad\qquad},\\
q_0(x) =& \underline{?\qquad\qquad},\\
p_{-1}(x) =& x^7,\\
p_0(x) =& x^6,\\
d_{-1} =& -1, d_0 = 0, z_0 = -1.
\end{aligned}
$$

(2 marks)

(c) To find the error locations by using the BerlekampMassey algorithm, we can get the following table:

Table 1: Calculation results of step (c)

i	$q_i - p_i$									d_i	z_i
-1	α^0	α^{10}	α^4	α^8	α^{13}	α^7	α^{11}	–	α^0	-1	
0	α^{10}	α^4	α^8	α^{13}	α^7	α^{11}	–	α^0		0	-1
1	α^8	α^6	α^8	α^{11}	α^9	–	α^0	α^{10}		0	0
2	$\underline{?}$	α^{14}	0	α^6	–	α^0	α^9			1	1
3	α^7	α^3	0	–	α^0	$\underline{?}$	α^5			1	2
4	0	0	–	α^0	α^{11}	α^7				2	3
5	0	–	α^0	α^{11}	α^7					3	3
6	–	α^0	α^{11}	α^7						4	3

(2 marks)

Finally, we obtain:

$$\sigma(x) = \underline{?\qquad\quad}.$$

(1 marks)

(d) According to $\sigma(x)$, we get the error location numbers as $\underline{?\ \ }$ and $\underline{?\ \ }$.

(2 marks)

(e) Solve the following function

$$\begin{pmatrix} 1 & 1 \\ \underline{?} & \underline{?} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \underline{?} \\ \alpha^4 \end{pmatrix}$$

and we get $b_1 = \underline{?\ }$ and $b_2 = \underline{?\ }$.

(5 marks)

The most likely error pattern is:

$$e = \underline{?\qquad\qquad}.$$

(1 marks)

Then

$$c = w + e = \underline{?\qquad\qquad}.$$

(1 marks)

Table 2: Construction of a $GF(2^4)$ field by $h(x) = 1 + x + x^4$

Codeword	Polynomial in x (mod $h(x)$)	Power of α
0000	0	–
1000	1	1
0100	x	α
0010	x^2	α^2
0001	x^3	α^3
1100	$1+x$	α^4
0110	$x+x^2$	α^5
0011	x^2+x^3	α^6
1101	$1+x+x^3$	α^7
1010	$1+x^2$	α^8
0101	$x+x^3$	α^9
1110	$1+x+x^2$	α^{10}
0111	$x+x^2+x^3$	α^{11}
1111	$1+x+x^2+x^3$	α^{12}
1011	$1+x^2+x^3$	α^{13}
1001	$1+x^3$	α^{14}

(Total 16 marks)

(Exam Total 105 marks)

(100%=100 marks)