Convolutional Codes and Viterbi Decoding

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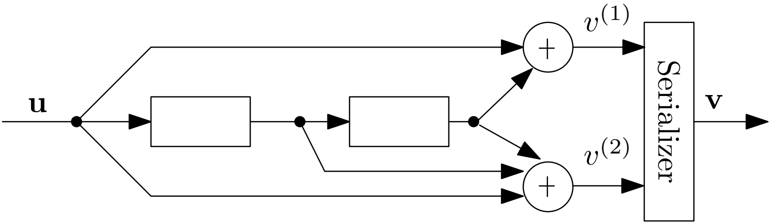
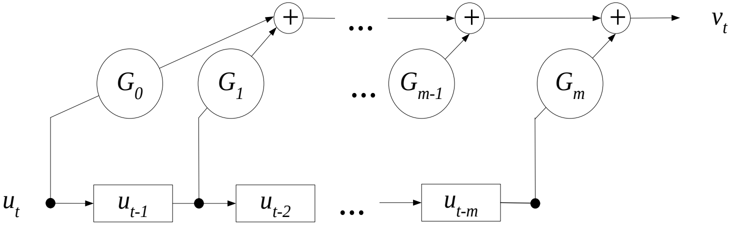
Outline

Introduction and Representations



$$\begin{cases} v_t^{(1)} = u_t + u_{t-2} \\ v_t^{(2)} = u_t + u_{t-1} + u_{t-2}. \end{cases}$$

Introduction and Representations



$$v_t = u_t G_0 + u_{t-1} G_1 + \cdots + u_{t-m} G_m$$

Introduction and Representations

$$\mathbf{v} = \mathbf{u}\mathbf{G},$$

where

$$\begin{pmatrix}
G_0 & G_1 & \dots & G_m & & & & & \\
 & G_0 & G_1 & \dots & G_m & & & & \\
 & & G_0 & G_1 & \dots & G_m & & & \\
 & & & G_0 & G_1 & \dots & G_m & & \\
 & & & & \vdots & \vdots & \vdots & \vdots & \\
 & & & & & \vdots & \vdots & \vdots & \vdots
\end{pmatrix}$$

Introduction and Representations

$$v^{(j)} = \sum_{i=1}^{k} u^{(i)} * \mathbf{g}_i^{(j)}, j = 1, 2, \ldots, n.$$

Example

$$\begin{cases} \mathbf{g}^{(1)} = (101) \\ \mathbf{g}^{(2)} = (111) \end{cases}$$

the composite generator sequence is

$$\mathbf{g} = 11\ 01\ 11.$$

Introduction and Representations

$$\begin{pmatrix} \mathbf{g}_1^{(1)}(D) & \mathbf{g}_1^{(2)}(D) & \dots & \mathbf{g}_1^{(n)}(D) \\ \mathbf{g}_2^{(1)}(D) & \mathbf{g}_2^{(2)}(D) & \dots & \mathbf{g}_2^{(n)}(D) \\ \vdots & \vdots & \vdots & \\ \mathbf{g}_k^{(1)}(D) & \mathbf{g}_k^{(2)}(D) & \dots & \mathbf{g}_k^{(n)}(D) \end{pmatrix}$$

Example

$$G(D) = [1 + D^2 \; 1 + D + D^2]$$

Definitions

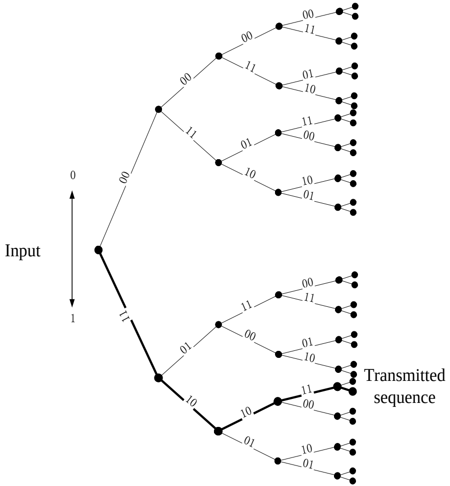
Constraint length

$$v_i = \max_{1 \le j \le n} [deg(\mathbf{g}_i^{(j)}(D))]$$

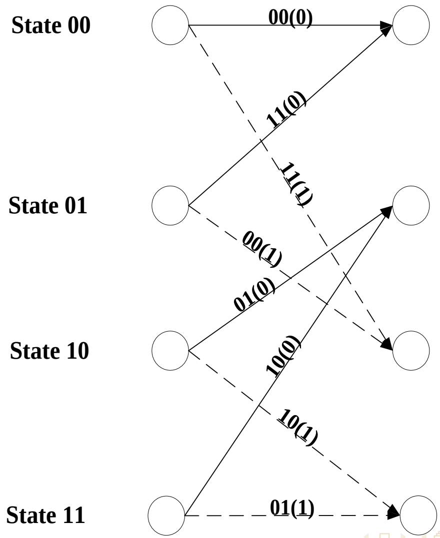
Overall Constraint length

$$v = \sum_{i=1}^{k} v_i$$

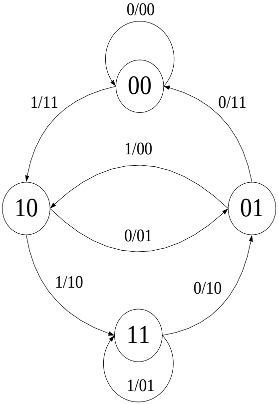
Tree Representation



Trellis Representation



State Representation



- Catastrophic Convolutional Code (Example 3.3)

Distance Properties

- Euclidean Distance
- Hamming Distance
- Free Distance
- Error Correcting Capability: $t_{free} = \left\lfloor \frac{d_{free}-1}{2} \right\rfloor$
- Distance Spectrum: $n(d_{free} + i)$

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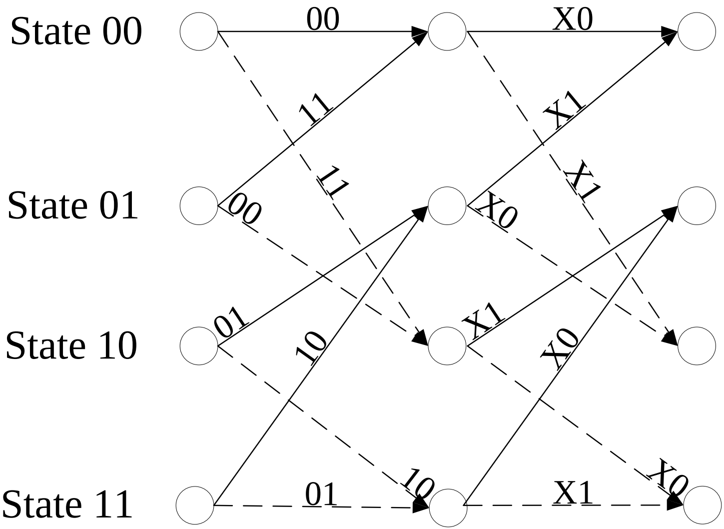
Punctured Codes



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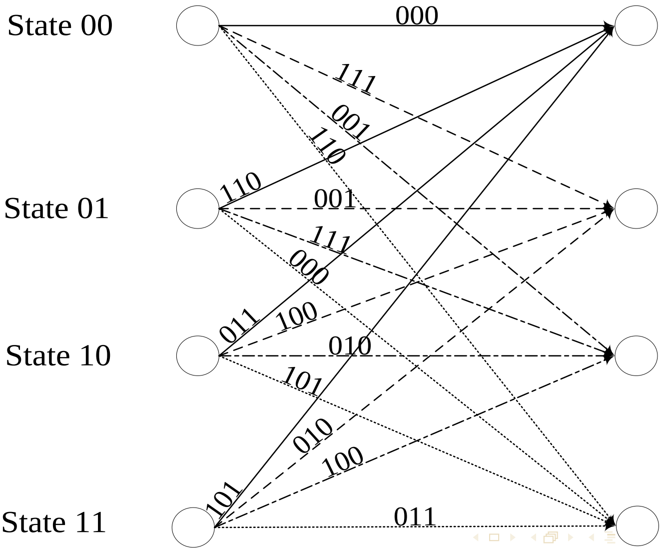
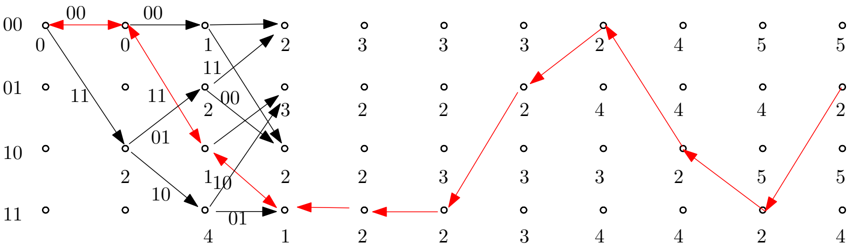
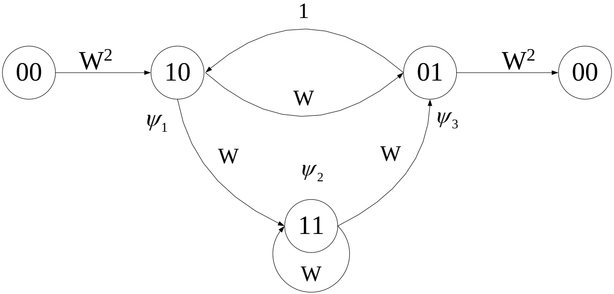


Viterbi Decoding



Received:	00	01	10	11	01	10	11	11	10	10
Decoded:	0	1	1	1	1	0	0	1	1	0

Signal Flow Chart



Upper Bounds of Burst Error Probability

$$P_B < \sum_{d=d_{free}}^{\infty} n_d \left(2\sqrt{\epsilon(1-\epsilon)}\right)^d = T(W)\Big|_{W=2\sqrt{\epsilon(1-\epsilon)}}$$