

Convolutional Codes and Viterbi Decoding

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October 20-21, 2010

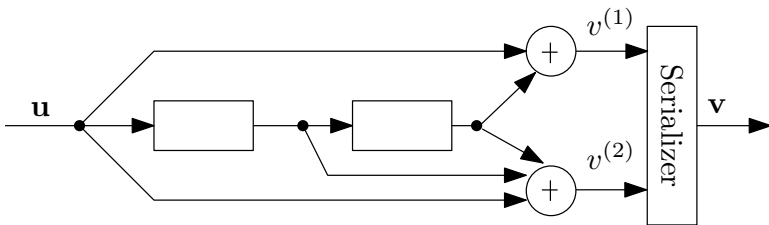


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- 1 Introduction
- 2 Basics
- 3 Graphical Representation of Convolutional Codes
- 4 Distance Properties
- 5 Punctured Codes, Rate-Compatible Encoder and Unequal Error Protection
- 6 Viterbi Decoding



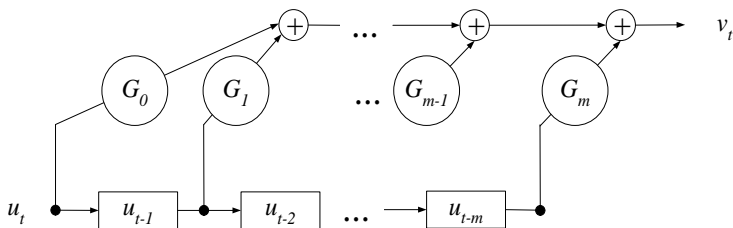
Introduction and Representations



$$\begin{cases} v_t^{(1)} = u_t + u_{t-2} \\ v_t^{(2)} = u_t + u_{t-1} + u_{t-2}. \end{cases}$$



Introduction and Representations



$$v_t = u_t G_0 + u_{t-1} G_1 + \dots + u_{t-m} G_m$$



Introduction and Representations

$$v^{(j)} = \sum_{i=1}^k u^{(i)} * \mathbf{g}_i^{(j)}, j = 1, 2, \dots, n.$$

Example

$$\begin{cases} \mathbf{g}^{(1)} = (101) \\ \mathbf{g}^{(2)} = (111) \end{cases}$$

the composite generator sequence is

$$\mathbf{g} = 11 \ 01 \ 11.$$



Introduction and Representations

$$\begin{pmatrix} \mathbf{g}_1^{(1)}(D) & \mathbf{g}_1^{(2)}(D) & \dots & \mathbf{g}_1^{(n)}(D) \\ \mathbf{g}_2^{(1)}(D) & \mathbf{g}_2^{(2)}(D) & \dots & \mathbf{g}_2^{(n)}(D) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{g}_k^{(1)}(D) & \mathbf{g}_k^{(2)}(D) & \dots & \mathbf{g}_k^{(n)}(D) \end{pmatrix}$$

Example

$$G(D) = [1 + D^2 \quad 1 + D + D^2]$$



Definitions

Constraint length

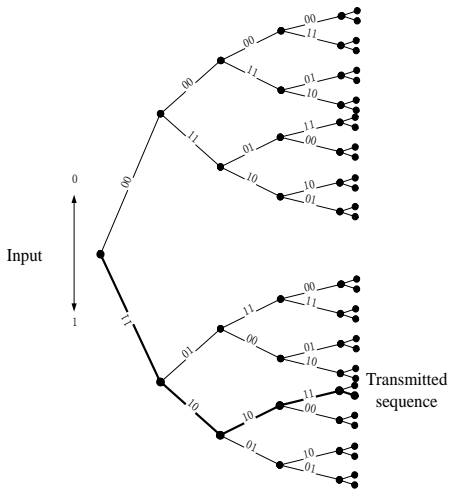
$$v_i = \max_{1 \leq j \leq n} [\deg(\mathbf{g}_i^{(j)}(D))]$$

Overall Constraint length

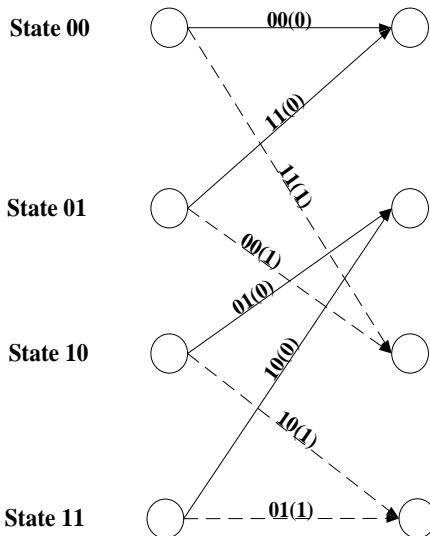
$$v = \sum_{i=1}^k v_i$$



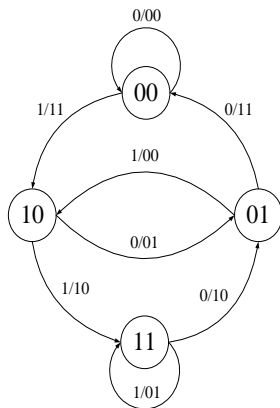
Tree Representation



Trellis Representation



State Representation



- Catastrophic Convolutional Code (Example 3.3)



Distance Properties

- Euclidean Distance
- Hamming Distance
- Free Distance
- Error Correcting Capability: $t_{free} = \left\lfloor \frac{d_{free}-1}{2} \right\rfloor$
- Distance Spectrum: $n(d_{free} + i)$



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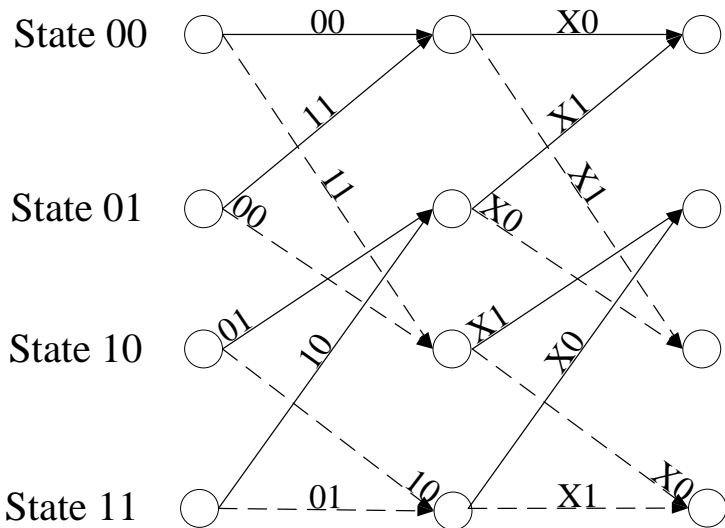


Distance Properties

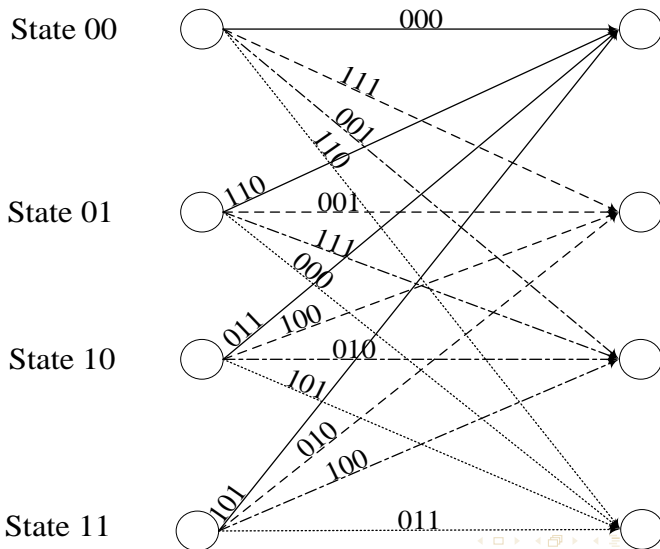
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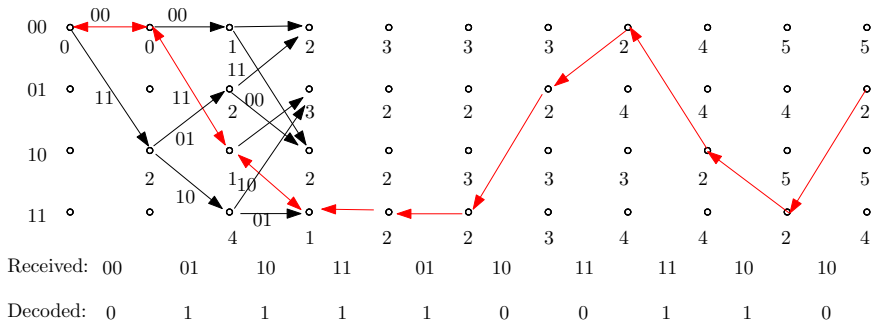
Punctured Codes



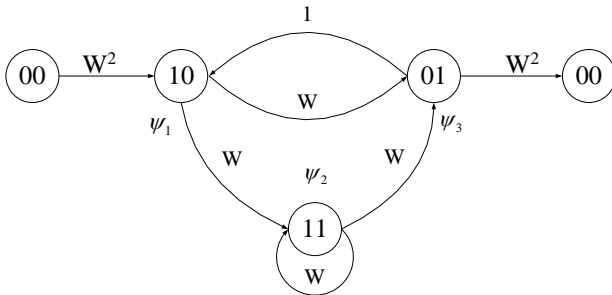
Punctured Codes



Viterbi Decoding



Signal Flow Chart



Upper Bounds of Burst Error Probability

$$P_B < \sum_{d=d_{free}}^{\infty} n_d \left(2\sqrt{\epsilon(1-\epsilon)} \right)^d = T(W)|_{W=2\sqrt{\epsilon(1-\epsilon)}}$$

