Digital Transmission through the Additive White Gaussian Noise Channel

ELEN 3024 - Communication Fundamentals

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Digital Transmission Through the AWGN Channel

Proakis and Salehi, “Communication Systems Engineering” (2nd Ed.), Chapter 7

Overview

7.5. Optimum Receiver for Digitally Modulated Signals in Additive White Gaussian Noise

Consider digital communication system that transmits digital information by use of any one of the M-ary signal waveforms

Input sequence to the modulator subdivided into k-bit blocks

Each of the $M = 2^k$ symbols is associated with a corresponding baseband signal waveform from set $\{s_m(t),\ m = 1, 2, \ldots, M\}$

Each signal is transmitted within the symbol interval $T \rightarrow$ consider transmission of information over interval $0 \leq t \leq T$

7.5. Optimum Receiver for Digitally Modulated Signals in Additive White Gaussian Noise

Channel corrupt signal by addition of AWGN (Fig. 7.30)

$$r(t) = s_m(t) + n(t),\ 0 \leq t \leq T$$

$n(t)$ sample function of the additive white Gaussian noise with power-spectral density $S_n(f) = \frac{N_0}{2}$ W/Hz

Based on the observation of $r(t)$ over the signal interval, we wish to design a receiver that is optimum in the sense that it minimizes the probability of making an error

7.5. Optimum Receiver for Digitally Modulated Signals in Additive White Gaussian Noise

Convenient to subdivide receiver into two parts:

- signal demodulator
- detector

Function of signal demodulator is to convert the received waveform $r(t)$ into an N-dimensional vector $\mathbf{r} = (r_1, r_2, \ldots, r_N) \to N$ dimension the transmitted signal waveforms

Function of detector is to decide which of the M possible signal waveforms was transmitted, based on observation of the vector $\mathbf{r}$

7.5. Optimum Receiver for Digitally Modulated Signals in Additive White Gaussian Noise

Two realizations of the signal demodulator described in following sections:

- Correlation-type demodulator
- Matched-filter type demodulator

Optimum detector that follows the signal demodulator is designed to minimize the probability of error

7.5.1. Correlation-Type Demodulator

Demodulator decomposes the received signal and noise into N-dimensional vectors

Signal $+$ noise $\rightarrow$ expanded into a series of linearly weighted orthonormal basis functions $\{\psi_n(t)\}$

It is assumed that the N basis functions $\{\psi_n(t)\}$ span the signal space $\rightarrow$ every $\{s_m(t)\}$ expressed as a weighted linear combination of $\{\psi_n(t)\}$

In case of the noise, the functions $\{\psi_n(t)\}$ do not span the noise space $\rightarrow$ shown that noise terms that fall outside the signal space are irrelevant to detection of signal

7.5.1. Correlation-Type Demodulator

Suppose received signal $r(t)$ is passed through a parallel bank of N cross correlators $\rightarrow$ compute projection of $r(t)$ onto the N basis functions $\{\psi_n(t)\}$ $\rightarrow$ Fig. 7.31

$$\begin{array}{rcl} \int_0^T r(t)\psi_k(t)dt & = & \int_0^T [s_m(t) + n(t)]\,\psi_k(t)dt \\ r_k & = & s_{mk} + n_k,\ k = 1, 2, \ldots, N \end{array}$$

where

$$s_{mk} = \int_0^T s_m(t)\psi_k(t)dt,\ k = 1, 2, \ldots, N$$

$$n_k = \int_0^T n(t)\psi_k(t)dt,\ k = 1, 2, \ldots, N$$

$$\therefore \mathbf{r} = \mathbf{s_m} + \mathbf{n}$$

7.5.1. Correlation-Type Demodulator

Signal now represented by the vector $\mathbf{s_m}$ with components $s_{mk},\ k = 1, 2, \ldots, N$.

$\{s_{mk}\}$ depend on which of the M signals was transmitted

Components of $\mathbf{n}$ i.e., $\{n_k\}$ are random variables that arise from the presence of the additive noise

7.5.1. Correlation-Type Demodulator

Can express received signal $r(t)$ in the interval $0 \leq t \leq T$ as

$$\begin{aligned} r(t) &= \textstyle\sum_{k=1}^{N} s_{mk}\psi_k(t) + \sum_{k=1}^{N} n_k\psi_k(t) + n'(t) \\ &= \textstyle\sum_{k=1}^{N} r_k\psi_k(t) + n'(t) \end{aligned}$$

Term $n'(t)'$:

$$n'(t) = n(t) - \sum_{k=1}^{N} n_k\psi_k(t)$$

$n'(t) \rightarrow$ zero-mean, Gaussian noise process that represents the difference between original noise process $n(t)$ and the part that corresponds to the projection of $n(t)$ onto basis functions $\{\psi_k(t)\}$

7.5.1. Correlation-Type Demodulator

$n'(t)$ irrelevant to the decision as to which signal was transmitted

$\Rightarrow$ decision of which symbol transmitted based entirely on the correlator output signal and noise components $r_k = s_{mk} + n_k$

Signals $\{s_m(t)\}$ deterministic $\rightarrow$ signal components are deterministic.

Noise components $\{n_k\}$ Gaussian $\rightarrow$ mean values

$$E[n_k] = \int_0^T E[n(t)]\psi_k(t)dt = 0, \; \forall \; k$$

Covariances are

$$E[n_k, m_k] \;\; = \;\; \int_0^T \int_0^T E[]$$

7.5.1. Correlation-Type Demodulator

7.5.2. Matched-Filter-Type Demodulator

Instead of using a bank of N correlators to generate the variables $\{r_k\}$, we may use a bank of N linear filters.

Assume that impulse responses of the N filters are:

$$h_k(t) = \psi_k(T - t), \;\; 0 \leq t \leq T$$

where $\psi_k(t)$ are the N basis functions and $h_k(t) = 0$ outside interval $0 \leq t \leq T$.

Output of these filters are

$$\begin{aligned} y_k(t) &= \int_0^t r(\tau) h_k(t - \tau) d\tau \\ &= \int_0^t r(\tau) \psi_k(T - t + \tau) d\tau, \;\; k = 1, 2, \ldots, N \end{aligned}$$

7.5.2. Matched-Filter-Type Demodulator

If we sample outputs of these filters at $t = T$, we obtain

$$y_k(T) = \int_0^T r(\tau)\psi_k(\tau)d\tau = r_k, \quad k = 1, 2, \ldots, N$$

Sampled outputs of the filters at time $t = T$ are exactly the same as the set of values $\{r_k\}$ obtained from the N linear correlators

7.5.2. Matched-Filter-Type Demodulator