Frequency Domain Analysis of Signals and Systems

ELEN 3024 - Communication Fundamentals

School of Electrical and Information Engineering, University of the Witwatersrand

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Amplitude Modulation

Proakis and Salehi, “Communication Systems Engineering” (2nd Ed.), Chapter 3

Overview

Power content of various AM modulation schemes

3.2.1.2 Double-Sideband Supressed Carrier AM

$$u(t) = A_c m(t) \cos(2\pi f_c t + \phi)$$

Assume phase of signal set to zero $\rightarrow$ power in signal is independent of phase

3.2.1.2 Double-Sideband Supressed Carrier AM

time-average autocorrelation function of $u(t)$

$$R_u(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u(t)u(t-\tau)\mathrm{dt}$$

3.2.1.2 Double-Sideband Supressed Carrier AM

time-average autocorrelation function of $u(t)$

$$
\begin{aligned}
R_u(\tau) &= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} u(t)u(t-\tau)\mathrm{dt} \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} A_c^2 m(t)m(t-\tau)\times \\
&\qquad \cos(2\pi f_c t)\cos(2\pi f_c(t-\tau))\mathrm{dt} \\
&= \frac{A_c^2}{2}\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} m(t)m(t-\tau)\times \\
&\qquad [\cos(4\pi f_c t - 2\pi f_c\tau) + \cos(2\pi f_c\tau)]\,\mathrm{dt} \\
&= \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c\tau)
\end{aligned}
$$

3.2.1.2 Double-Sideband Supressed Carrier AM

Used the fact that:

$$\lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c \tau)\mathrm{dt} = 0$$

Because

$$\begin{aligned}
&\int_{-\infty}^{\infty} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c\tau)\mathrm{dt} \\
&= \int_{-\infty}^{\infty} \mathcal{F}\left[m(t-\tau)\right]\left\{\mathcal{F}\left[m(t)\cos(4\pi f_c t - 2\pi f_c\tau)\right]\right\}^* \mathrm{df} \\
&= \int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f)\left[\frac{M(f-2f_c)e^{-j2\pi f_c t}}{2} + \frac{M(f+2f_c)e^{j2\pi f_c t}}{2}\right]^* \mathrm{df} \\
&= 0
\end{aligned}$$

3.2.1.2 Double-Sideband Supressed Carrier AM

$$\int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f) \left[\frac{M(f-2f_c)e^{-j2\pi f_c t}}{2} + \frac{M(f+2f_c)e^{j2\pi f_c t}}{2} \right]^* \mathrm{df} = 0$$

Why?

3.2.1.2 Double-Sideband Supressed Carrier AM

$$\int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f) \left[\frac{M(f-2f_c)e^{-j2\pi f_c t}}{2} + \frac{M(f+2f_c)e^{j2\pi f_c t}}{2} \right]^* \mathrm{df} = 0$$

Why?

$M(f)$ limited to the frequency band $[-W, W]$ and $W \ll f_c$, therefore no frequency overlap between $M(f)$ and $M(f \pm 2f_c)$

3.2.1.2 Double-Sideband Supressed Carrier AM

Fourier transform on both sides of:

$$\begin{aligned} \mathcal{F}(R_u(\tau)) &= \mathcal{F}(\tfrac{A_c^2}{2} R_m(\tau) \cos(2\pi f_c \tau)) \\ S_u(f) &= \tfrac{A_c^2}{4} [S_m(f - f_c) + S_m(f + f_c)] \end{aligned}$$

$\Rightarrow$ power-spectral density of DSB-SC signal is the power-spectral density of the message shifted upward and downward by f_c and scaled by $A_c^2/4$.

3.2.1.2 Double-Sideband Supressed Carrier AM

To obtain total power in modulated signal

- Substitute $\tau = 0$ in time-average autocorrelation function
- integrate power-spectral density of modulated signal

$$
\begin{aligned}
P_u &= \frac{A_c^2}{2} R_m(\tau) \cos(2\pi f_c \tau)|_{\tau=0} \\
&= \frac{A_c^2}{2} R_m(0) \\
&= \frac{A_c^2}{2} P_m
\end{aligned}
$$

3.2.1.2 Double-Sideband Supressed Carrier AM

Example 3.2.2

3.2.2.2 Conventional Amplitude Modulation

Conventional AM signal similar to DSB when $m(t)$ is substituted with $1 + am_n(t)$

$$P_u = \frac{A_c^2}{2} P_m$$

P_m power in the message signal.

3.2.2.2 Conventional Amplitude Modulation

For AM DSB FC:

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (1 + am_n(t))^2 \mathrm{dt}$$
$$\lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (1 + a^2 m_n^2(t)) \mathrm{dt}$$

Assuming average of $m_n(t) = 0$.

$$P_m = 1 + a^2 P_{m_n}$$

3.2.2.2 Conventional Amplitude Modulation

Hence

$$P_u = \frac{A_c^2}{2} + \frac{A_c^2}{2} a^2 P_{m_n}$$

First component $\rightarrow$ carrier

Second component $\rightarrow$ information carrying component.