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University of the Witwatersrand, Johannesburg

Course or topic No(s) $\square$

Course or topic name(s)
Paper Number \& title

Examination/Test* to be
held during month(s) of
(*delete as applicable)
Year of Study
(Art \& Sciences leave blank)

Degrees/Diplomas for which
this course is prescribed
(BSc (Eng) should indicate which branch)

Faculty/ies presenting candidates

Internal examiners
and telephone
number(s)

External examiner(s)

Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)

Time allowance

Dr. K Ouahada


| Course <br> Nos | ELEN3024 | Hours | Three |
| :---: | :---: | :---: | :---: |

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            Answer ALL questions.
Type '2' Examination - Written A4 permitted.
            Show all workings.
        Total marks: 115 - Full marks: }10
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Internal Examiners or Heads of Department are requested to sign the declaration overleaf

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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.

## Question 1

Consider the following analogue communication system. When provided with a single tone

$$
f_{m}=3 \cos (2 \pi 2000 t)[\mathrm{V}]
$$

as input, the output of the system as displayed on a spectrum analyser is depicted in Fig. 1. Refer to the table of Bessel functions at the back of the paper.


Figure 1: Output spectrum
(a) Assuming that the system is an AM modulator, determine the modulation factor, frequency of the carrier frequency and total transmit power.
( 5 marks)
(b) Assuming that the system is an FM modulator, determine the modulation factor, unmodulated carrier (frequency as well as amplitude) and total transmit power.
( 5 marks)
(c) Devise two independent tests to identify the system as either an AM or FM modulator. Show the exact steps as well as the expected outcome for each step. You have only access to the input signal, and can only change the amplitude and frequency thereof.

## Question 2

Refer to the modulator depicted in Fig. 2. In the modulator, we have made use of the following terminology.

- A subscript 'b' denotes binary data, e.g. $x_{b}$. The binary data can either be a logical 0 or logical 1, where the AND operation is depicted by "." and the OR operation is depicted by "+".
- A subscript 'a' denotes antipodal data, e.g. $x_{a}$. The binary data is encoded with a non-return-to-zero encoder, i.e., a binary 1 is mapped to a 1 , and a binary 0 is mapped to -1
- Here $x_{b} \times d$ denotes a binary number $x_{b}$ multiplied by the constant $d(d \in \mathbb{R})$; the result is $d$ when $x_{b}=1$, and 0 when $x_{b}=0$.


Figure 2: Modulator
(a) Sketch the corresponding constellation for the modulator assuming that the 3-tuple $\left(a_{b}, b_{b}, c_{b}\right)$ can take on any combination of 1 s and 0 s . Clearly indicate the distances between constellation points, as well as the 3 -tuple corresponding to each constellation point.
(b) Compare the maximum allowable peak error for the constellation determined in (a) to the maximum allowable peak error for the corresponding $M$-PSK constellation. Also compare the average power for each constellation, expressing the ratio of the two in decibels. Assume that for both systems the maximum amplitude is $A$.
(c) Show the optimal decision boundaries for the constellation in (a).
(d) Determine the minimum symbol rate necessary if the desired data rate is $30 \mathrm{Mbits} / \mathrm{s}$.
(e) Is the mapping of bits to symbols optimal for the modulator of Fig. 2? If yes, motivate your answer. If not, show how the mapping can be made optimal.
(f) Which system of the above two (i.e., system depicted in (a), or $M$-PSK) would you propose to be used? Motivate your answer.

## Question 3

Refer to the pairs of signals in Signal set I (Fig. 3) and Signal set II (Fig. 4) in order to answer the following questions.


Figure 3: Signal set I


Figure 4: Signal set II
(a) Show that the signals $\psi_{1}(t)$ and $\psi_{2}(t)$ (Signal set I) are orthogonal over one symbol period $T$.
( 2 marks)
(b) Express the signals of Signal set II in terms of the signals $\psi_{1}(t)$ and $\psi_{2}(t)$ (write equations). Sketch a "constellation diagram" where $\psi_{1}(t)$ represents the x-axis and $\psi_{2}(t)$ represents the y -axis.
( 6 marks)
(c) By observing the constellation diagram only, is the signal pair of Signal set II orthogonal or not?
( 2 marks)
(d) Verify your answer in (c).
( 2 marks)
(e) Develop a communication system that can transmit 8 bits per symbol, which uses the signal pair of Signal set II. (Assume perfect symbol synchronisation has been achieved.) Sketch the signals used, a block diagram of the transmitter and a block diagram of the receiver.

## Question 4

Consider the 16-QAM constellation with alphabet $\mathcal{A}=\{ \pm c \pm j c, \pm c \pm j 3 c, \pm 3 c \pm j c, \pm 3 c \pm$ $j 3 c\}$ depicted in Fig. 5. Derive expressions for both the symbol error probability and symbol erasure probability for this constellation. Assume that an erasure occurs whenever the received symbol falls within any of the shaded regions of width $b$. Assume further that $b=m \cdot d$, where $m \in[0,1]$, and $d$ the distance indicated in Fig. 5.


Figure 5: 16-QAM with errors and erasures

## Question 5

Consider a baseband PAM system with output as depicted in Fig. 6.
(a) Show that when the sampled pulse shape $g(k T)=\delta_{k}$, no intersymbol interference is present.
(b) Assuming that the symbol period $T=1$, demodulate the pulse train depicted in Fig. 6, where a timing offset is introduced. Show the individual pulses used for transmission and calculate the timing offset.
(c) For this particular PAM system, how many bits are represented per symbol?
(d) What is the effect of the raised-cosine filter roll-off factor $\alpha$ on the system?
( 3 marks)


Figure 6: Pulse train
( Total 20 marks)
( Exam Total: 115 marks)
(Full marks $=100$ marks)
Table 1: Bessel functions of the first kind, $J_{n}(m)$

| $\beta$ | $J_{0}$ | $J_{1}$ | $J_{2}$ | $J_{3}$ | $J_{4}$ | $J_{5}$ | $J_{6}$ | $J_{7}$ | $J_{8}$ | $J_{9}$ | $J_{10}$ | $J_{11}$ | $J_{12}$ | $J_{13}$ | $J_{14}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.25 | 0.98 | 0.12 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.5 | 0.94 | 0.24 | 0.03 |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.0 | 0.77 | 0.44 | 0.11 | 0.02 |  |  |  |  |  |  |  |  |  |  |  |
| 1.5 | 0.51 | 0.56 | 0.23 | 0.06 | 0.01 |  |  |  |  |  |  |  |  |  |  |
| 2.0 | 0.22 | 0.58 | 0.35 | 0.13 | 0.03 |  |  |  |  |  |  |  |  |  |  |
| 2.4 | 0 | 0.52 | 0.43 | 0.20 | 0.06 | 0.02 |  |  |  |  |  |  |  |  |  |
| 2.5 | -0.05 | 0.50 | 0.45 | 0.22 | 0.07 | 0.02 | 0.01 |  |  |  |  |  |  |  |  |
| 3.0 | -0.26 | 0.34 | 0.49 | 0.31 | 0.13 | 0.04 | 0.01 |  |  |  |  |  |  |  |  |
| 4.0 | -0.40 | -0.07 | 0.36 | 0.43 | 0.28 | 0.13 | 0.05 | 0.02 |  |  |  |  |  |  |  |
| 5.0 | -0.18 | -0.33 | 0.05 | 0.36 | 0.39 | 0.26 | 0.13 | 0.05 | 0.02 |  |  |  |  |  |  |
| 5.45 | 0 | -0.34 | -0.12 | 0.26 | 0.40 | 0.32 | 0.19 | 0.09 | 0.03 | 0.01 |  |  |  |  |  |
| 6.0 | 0.15 | -0.28 | -0.24 | 0.11 | 0.36 | 0.36 | 0.25 | 0.13 | 0.06 | 0.02 |  |  |  |  |  |
| 7.0 | 0.30 | 0.00 | -0.30 | -0.17 | 0.16 | 0.35 | 0.34 | 0.23 | 0.13 | 0.06 | 0.02 |  |  |  |  |
| 8.0 | 0.17 | 0.23 | -0.11 | -0.29 | -0.10 | 0.19 | 0.34 | 0.32 | 0.22 | 0.13 | 0.06 | 0.03 |  |  |  |
| 8.65 | 0 | 0.27 | 0.06 | -0.24 | -0.23 | 0.03 | 0.26 | 0.34 | 0.28 | 0.18 | 0.10 | 0.05 | 0.02 |  |  |
| 9.0 | -0.09 | 0.25 | 0.14 | -0.18 | -0.27 | -0.06 | 0.20 | 0.33 | 0.31 | 0.21 | 0.12 | 0.06 | 0.03 | 0.01 |  |
| 10.0 | -0.25 | 0.05 | 0.25 | 0.06 | -0.22 | -0.23 | -0.01 | 0.22 | 0.32 | 0.29 | 0.21 | 0.12 | 0.06 | 0.03 | 0.01 |

