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University of the Witwatersrand, Johannesburg

Course or topic No(s) $\square$

Course or topic name(s)
Paper Number \& title

Examination/Test* to be
held during month(s) of
(*delete as applicable)
Year of Study
(Art \& Sciences leave blank)

Degrees/Diplomas for which
this course is prescribed
(BSc (Eng) should indicate which branch)

Faculty/ies presenting candidates

Internal examiners
and telephone
number(s)

External examiner(s)

Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)

Time allowance

Dr. K Ouahada


| Course <br> Nos | ELEN3024 | Hours | Three |
| :---: | :---: | :---: | :---: |

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            Answer ALL questions.
Type '2' Examination - Written A4 permitted.
            Show all working.
        Total marks: 115 - Full marks 100
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Internal Examiners or Heads of Department are requested to sign the declaration overleaf

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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.

## Question 1

Refer to the modulator depicted in Fig. 1.
(a) Sketch the output $v_{\text {out }}$ in the time domain when a square wave with frequency $f_{c}$ is used to modulate a sinusoidal signal $m(t)=a \cos 2 \pi f_{m} t$, under the condition that $f_{m} \ll f_{c}$.
(b) Sketch what happens in the frequency domain, assuming that all appropriate filtering has been done to ensure that the circuit operates as a proper modulator.
( 2 marks)
(c) Assuming that the message signal, $m(t)=a \cos 2 \pi f_{m} t$, determine the power-spectral density of the modulated signal, the power in the modulated signal, and the power in each of the sidebands. (Base your calculations on the fact that the effective carrier is given as $A_{c} \cos 2 \pi f_{c} t$ )


Figure 1: Modulator

## Question 2

The message signal $m(t)=10 \operatorname{sinc}(400 t)$ frequency modulates the carrier $c(t)=100 \cos 2 \pi f_{c} t$ to give $u(t)$. The modulation index is 6 .
(a) Write an expression for the modulated signal $u(t)$.
(b) What is the maximum frequency deviation of the modulated signal?
(c) What is the power content of the modulated signal?
(d) Find the bandwidth of the modulated signal.

## Question 3

Determine under which conditions the following two signals can be used as basis functions for FSK transmission.

$$
\begin{aligned}
& u_{1}(t)=\sqrt{\frac{2 \mathcal{E}_{b}}{T_{b}}} \cos 2 \pi f_{1} t, 0 \leq t \leq T_{b} \\
& u_{2}(t)=\sqrt{\frac{2 \mathcal{E}_{b}}{T_{b}}} \cos 2 \pi f_{2} t, 0 \leq t \leq T_{b}
\end{aligned}
$$

## Question 4

For a receiver based on the minimum-distance criterion:
(a) Show that the cost function

$$
J=\int_{-\infty}^{\infty}|r(t)-a h(t)|^{2} d t
$$

can be reduced to

$$
J=E_{h}\left|\frac{y}{E_{h}}-a\right|^{2}-\frac{|y|^{2}}{E_{h}}+E_{r} .
$$

(b) Explain the general principle of operation.

## Question 5

Consider a baseband PAM system with output as depicted in Fig. 2.
(a) Show that when the sampled pulse shape $g(k T)=\delta_{k}$, no intersymbol interference is present.
( 5 marks)
(b) Assuming that the symbol period $T=1$, demodulate the pulse train depicted in Fig. 2, and show the individual pulses used for transmission.
(c) For this particular PAM system, how many bits are represented per symbol?
(d) What is the effect of the roll-off factor $\alpha$ on the system?


Figure 2: Pulse train

## Question 6

(a) Consider the two octal signal point constellations depicted in Fig. 3. Compare the maximum allowable peak error for the two constellations, as well as the average power for each constellation, assuming that for both systems the maximum amplitude is $A$. For the constellation of Fig. 3a, all the nearest-neighbour signal points are separated in distance by $d_{1}$ units, whilst in Fig. 3b the minimum distance between constellation points is $d_{2}$.
(b) Show the optimal decision boundaries for the constellation of Fig. 3a.
( 5 marks)
(c) Show the optimal decision boundaries for the constellation of Fig. 3b.
(d) Determine the minimum symbol rate necessary if the desired data rate is $90 \mathrm{Mbits} / \mathrm{s}$.
(e) Is it possible to assign 3 data bits to each point of the signal constellation such that the nearest (adjacent) points differ in only one position for Fig. 3a? If so, sketch the solution; if not, explain why not.
( 3 marks)
(f) Is it possible to assign 3 data bits to each point of the signal constellation such that the nearest (adjacent) points differ in only one position for Fig. 3b? If so, sketch the solution; if not, explain why not.
(g) Which system would you propose to be used? Motivate your answer.
( 2 marks)

(a) 8-QAM System A

(b) 8-QAM System B

Figure 3: Two alternative octal signal point constellations

## Question 7

Consider the 16-QAM constellation with alphabet $\mathcal{A}=\{ \pm c \pm j c, \pm c \pm j 3 c, \pm 3 c \pm j c, \pm 3 c \pm$ $j 3 c\}$. Sketch the corresponding constellation and determine an approximate expression for the symbol error rate.
Table 1: Bessel functions of the first kind, $J_{n}(m)$

| $\beta$ | $J_{0}$ | $J_{1}$ | $J_{2}$ | $J_{3}$ | $J_{4}$ | $J_{5}$ | $J_{6}$ | $J_{7}$ | $J_{8}$ | $J_{9}$ | $J_{10}$ | $J_{11}$ | $J_{12}$ | $J_{13}$ | $J_{14}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.25 | 0.98 | 0.12 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.5 | 0.94 | 0.24 | 0.03 |  |  |  |  |  |  |  |  |  |  |  |  |
| 1.0 | 0.77 | 0.44 | 0.11 | 0.02 |  |  |  |  |  |  |  |  |  |  |  |
| 1.5 | 0.51 | 0.56 | 0.23 | 0.06 | 0.01 |  |  |  |  |  |  |  |  |  |  |
| 2.0 | 0.22 | 0.58 | 0.35 | 0.13 | 0.03 |  |  |  |  |  |  |  |  |  |  |
| 2.4 | 0 | 0.52 | 0.43 | 0.20 | 0.06 | 0.02 |  |  |  |  |  |  |  |  |  |
| 2.5 | -0.05 | 0.50 | 0.45 | 0.22 | 0.07 | 0.02 | 0.01 |  |  |  |  |  |  |  |  |
| 3.0 | -0.26 | 0.34 | 0.49 | 0.31 | 0.13 | 0.04 | 0.01 |  |  |  |  |  |  |  |  |
| 4.0 | -0.40 | -0.07 | 0.36 | 0.43 | 0.28 | 0.13 | 0.05 | 0.02 |  |  |  |  |  |  |  |
| 5.0 | -0.18 | -0.33 | 0.05 | 0.36 | 0.39 | 0.26 | 0.13 | 0.05 | 0.02 |  |  |  |  |  |  |
| 5.45 | 0 | -0.34 | -0.12 | 0.26 | 0.40 | 0.32 | 0.19 | 0.09 | 0.03 | 0.01 |  |  |  |  |  |
| 6.0 | 0.15 | -0.28 | -0.24 | 0.11 | 0.36 | 0.36 | 0.25 | 0.13 | 0.06 | 0.02 |  |  |  |  |  |
| 7.0 | 0.30 | 0.00 | -0.30 | -0.17 | 0.16 | 0.35 | 0.34 | 0.23 | 0.13 | 0.06 | 0.02 |  |  |  |  |
| 8.0 | 0.17 | 0.23 | -0.11 | -0.29 | -0.10 | 0.19 | 0.34 | 0.32 | 0.22 | 0.13 | 0.06 | 0.03 |  |  |  |
| 8.65 | 0 | 0.27 | 0.06 | -0.24 | -0.23 | 0.03 | 0.26 | 0.34 | 0.28 | 0.18 | 0.10 | 0.05 | 0.02 |  |  |
| 9.0 | -0.09 | 0.25 | 0.14 | -0.18 | -0.27 | -0.06 | 0.20 | 0.33 | 0.31 | 0.21 | 0.12 | 0.06 | 0.03 | 0.01 |  |
| 10.0 | -0.25 | 0.05 | 0.25 | 0.06 | -0.22 | -0.23 | -0.01 | 0.22 | 0.32 | 0.29 | 0.21 | 0.12 | 0.06 | 0.03 | 0.01 |

