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University of the Witwatersrand, Johannesburg

Course or topic No(s)

## ELEN3015

Course or topic name(s)
Paper Number \& title

Examination/Test* to be
held during month(s) of
(*delete as applicable)

Year of Study
(Art \& Sciences leave blank)

Degrees/Diplomas for which
this course is prescribed
(BSc (Eng) should indicate which branch)

Faculty/ies presenting candidates

Data and Information Management 2011/4/8 CM5

April 2011
$\square$
B.Sc (Eng) Elec.

## Engineering

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| Course <br> Nos | ELEN3015 | Hours | 1.5 |
| :---: | :---: | :---: | :---: |

Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

Answer $A L L$ questions.
Type '2' Examination.
Total marks: 47 - Full marks: 45

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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.

## Question 1

Given a (7, 4) Hamming code,
(a) If we add one more parity-check bit, which is the binary-sum of all the existing 7 bits, what is the parity-check matrix of the new code?
(b) What is the minimum Hamming distance of the new code? Prove it.

## Question 2

For a 3-error-correcting $(15,9)$ Reed-Solomon code with the generator polynomial

$$
\begin{aligned}
g(x) & =(x+1)(x+\alpha)\left(x+\alpha^{2}\right) \ldots\left(x+\alpha^{5}\right) \\
& =1+\alpha^{4} x+\alpha^{2} x^{2}+\alpha x^{3}+\alpha^{12} x^{4}+\alpha^{9} x^{5}+x^{6}
\end{aligned}
$$

Suppose the received word is $10 \alpha^{2} \alpha \alpha^{12} \alpha^{9} \alpha^{8} 00000000$, complete the following decoding process by using the Berlekamp-Massey algorithm:

The received sequence can be written in the polynomial

$$
w(x)=
$$

$\qquad$
(a) We get the syndrome polynomials as:

$$
\begin{aligned}
& s_{0}=w\left(\alpha^{0}\right)=? \\
& s_{1}=w\left(\alpha^{1}\right)=\alpha^{4}, \\
& s_{2}=w\left(\alpha^{2}\right)=\alpha^{8}, \\
& s_{3}=w\left(\alpha^{3}\right)=\alpha^{13}, \\
& s_{4}=w\left(\alpha^{4}\right)=\alpha^{7}, \\
& s_{5}=w\left(\alpha^{5}\right)=\alpha^{11}
\end{aligned}
$$

(b)

$$
\begin{aligned}
q_{-1}(x) & = \\
q_{0}(x) & = \\
p_{-1}(x) & =x^{7}, \\
p_{0}(x) & =x^{6}, \\
d_{-1} & =-1, d_{0}=0, z_{0}=-1 .
\end{aligned}
$$

(c) Preceding the step 3, we can get the following table:

Table 1: Calculation results of step (c)

| $i$ | $q_{i}-p_{i}$ |  |  |  |  |  |  |  | $d_{i}$ | $z_{i}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | $\alpha^{0}$ | $\alpha^{10}$ | $\alpha^{4}$ | $\alpha^{8}$ | $\alpha^{13}$ | $\alpha^{7}$ | $\alpha^{11}$ | - | $\alpha^{0}$ | -1 |  |
| 0 | $\alpha^{10}$ | $\alpha^{4}$ | $\alpha^{8}$ | $\alpha^{13}$ | $\alpha^{7}$ | $\alpha^{11}$ | - | $\alpha^{0}$ |  | 0 | -1 |
| 1 | $\alpha^{8}$ | $\alpha^{6}$ | $\alpha^{8}$ | $\alpha^{11}$ | $\alpha^{9}$ | - | $\alpha^{0}$ | $\alpha^{10}$ |  | 0 | 0 |
| 2 | $?$ | $\alpha^{14}$ | 0 | $\alpha^{6}$ | - | $\alpha^{0}$ | $\alpha^{9}$ |  |  | 1 | 1 |
| 3 | $\alpha^{7}$ | $\alpha^{3}$ | 0 | - | $\alpha^{0}$ | $?$ | $\alpha^{5}$ |  |  | 1 | 2 |
| 4 | 0 | 0 | - | $\alpha^{0}$ | $\alpha^{11}$ | $\alpha^{7}$ |  |  |  | 2 | 3 |
| 5 | 0 | - | $\alpha^{0}$ | $\alpha^{11}$ | $\alpha^{7}$ |  |  |  |  | 3 | 3 |
| 6 | - | $\alpha^{0}$ | $\alpha^{11}$ | $\alpha^{7}$ |  |  |  |  |  | 4 | 3 |

Finally, we obtain:

$$
\sigma(x)=
$$

$\qquad$
(d) According to $\sigma(x)$, we get the error location numbers as $\qquad$ and $\qquad$
(e) Solve the following function

$$
\left(\begin{array}{ll}
1 & 1 \\
- & -
\end{array}\right)\binom{b_{1}}{b_{2}}=\binom{-}{\alpha^{4}}
$$

and we get $b_{1}={ }_{-}$and $b_{2}=\ldots$.
The most likely error pattern is:

$$
e=
$$

Then

$$
c=w+e=
$$

## Question 3

Alice and Bob wish to communicate securely over an open channel using a columnar transposition cipher scheme with the column width no more than 8. Eva eavesdrops a ciphertext as follows:
ddh rat snt ode srh udo nla ode bqa oef fsy tun tif wfa lig mna ior ece eti zre akr
(a) Show the method to cryptanalyze the ciphertext by using the bigram.
(b) Show the sums of frequency of different possible solutions.
(c) Show the most likely plaintext.
$\left(\begin{array}{llllll}\text { th } & 1.52 \% & \text { en } & 0.55 \% & \text { ng } & 0.18 \% \\ h e & 1.28 \% & \text { ed } & 0.53 \% & \text { of } & 0.16 \% \\ \text { in } & 0.94 \% & \text { to } & 0.52 \% & \text { al } & 0.09 \% \\ \text { er } & 0.94 \% & \text { it } & 0.50 \% & \text { de } & 0.09 \% \\ \text { an } & 0.82 \% & \text { ou } & 0.50 \% & \text { se } & 0.08 \% \\ \text { re } & 0.68 \% & \text { ea } & 0.47 \% & \text { le } & 0.08 \% \\ n d & 0.63 \% & \text { hi } & 0.46 \% & \text { sa } & 0.06 \% \\ \text { at } & 0.59 \% & \text { is } & 0.46 \% & \text { si } & 0.05 \% \\ \text { on } & 0.57 \% & \text { or } & 0.43 \% & a r & 0.04 \% \\ n t & 0.56 \% & \text { ti } & 0.34 \% & \text { ve } & 0.04 \% \\ h a & 0.56 \% & \text { as } & 0.33 \% & \text { ra } & 0.04 \% \\ \text { es } & 0.56 \% & \text { te } & 0.27 \% & l d & 0.02 \% \\ \text { st } & 0.55 \% & \text { et } & 0.19 \% & u r & 0.02 \%\end{array}\right)$
( Total 15 marks)
( Exam Total 47 marks)
( $100 \%=45$ marks)

