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University of the Witwatersrand, Johannesburg

Course or topic No(s) $\square$

Course or topic name(s)
Paper Number \& title

Examination/Test* to be
held during month(s) of
(*delete as applicable)
Year of Study
(Art \& Sciences leave blank)

Degrees/Diplomas for which
this course is prescribed
(BSc (Eng) should indicate which branch)

Faculty/ies presenting candidates

Internal examiners
and telephone
number(s)

External examiner(s)

Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)

Time allowance
Dr. W. A. Clarke


| Course <br> Nos | ELEN3015 | Hours | One |
| :---: | :---: | :---: | :---: |

Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

Answer $A L L$ questions.
Type '2' Examination.

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

1. As the Internal Examiner/Head of Department, I certify that this question paper is in final form, as approved by the External Examiner, and is ready for reproduction.
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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.

## Question 1

What is ciphertext stealing? Explain how ciphertext stealing can be implemented in Electronic codebook mode (make use of sketches).
(Total 5 marks)

## Question 2

Refer to the algorithm depicted in Fig. 1 and the key schedule of Table 1.


Figure 1: Algorithm
(a) Identify the cryptographic system.
(b) Determine the output of the top left operation if $K_{1}=A B 78_{16}$ and the other input is $1 A B 8_{16}$.

Table 1: Key Schedule - Decryption

| Round | Subkeys |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1st | $Z_{1}^{(9)-1}$ | $-Z_{2}^{(9)}$ | $-Z_{3}^{(9)}$ | $Z_{4}^{(9)-1}$ | $Z_{5}^{(8)}$ | $Z_{6}^{(8)}$ |
| 2nd | $Z_{1}^{(8)-1}$ | $-Z_{3}^{(8)}$ | $-Z_{2}^{(8)}$ | $Z_{4}^{(8)-1}$ | $Z_{5}^{(7)}$ | $Z_{6}^{(7)}$ |
| 3rd | $Z_{1}^{(7)-1}$ | $-Z_{3}^{(7)}$ | $-Z_{2}^{(7)}$ | $Z_{4}^{(7)-1}$ | $Z_{5}^{(6)}$ | $Z_{6}^{(6)}$ |
| 4th | $Z_{1}^{(6)-1}$ | $-Z_{3}^{(6)}$ | $-Z_{2}^{(6)}$ | $Z_{4}^{(6)-1}$ | $Z_{5}^{(5)}$ | $Z_{6}^{(5)}$ |
| 5th | $Z_{1}^{(5)-1}$ | $-Z_{3}^{(5)}$ | $-Z_{2}^{(5)}$ | $Z_{4}^{(5)-1}$ | $Z_{5}^{(4)}$ | $Z_{6}^{(4)}$ |
| 6th | $Z_{1}^{(4)-1}$ | $-Z_{3}^{(4)}$ | $-Z_{2}^{(4)}$ | $Z_{4}^{(4)-1}$ | $Z_{5}^{(3)}$ | $Z_{6}^{(3)}$ |
| 7th | $Z_{1}^{(3)-1}$ | $-Z_{3}^{(3)}$ | $-Z_{2}^{(3)}$ | $Z_{4}^{(3)-1}$ | $Z_{5}^{(2)}$ | $Z_{6}^{(2)}$ |
| 8th | $Z_{1}^{(2)-1}$ | $-Z_{3}^{(2)}$ | $-Z_{2}^{(2)}$ | $Z_{4}^{(2)-1}$ | $Z_{5}^{(1)}$ | $Z_{6}^{(1)}$ |
| Last | $Z_{1}^{(1)-1}$ | $-Z_{2}^{(1)}$ | $-Z_{3}^{(1)}$ | $Z_{4}^{(1)-1}$ |  |  |

(c) Given that $K_{2}^{(9)}=2 C 1 B_{16}$, determine the value of $K_{2}$ for the first round of decryption.
(d) Given the key (in the form LSB . . . MSB):

$$
\begin{aligned}
& 1001101011101001011101110100101011101000101010101101000110111001 \\
& 1001100110010111001110101110100111000011111110010101010011010110
\end{aligned}
$$


( 4 marks)

## Question 3

Alice and Bob wish to communicate securely over an open channel using a public-key scheme. They decide to use the RSA algorithm.
(a) Using the Solovay-Strassen test, determine if the number 11131 is prime, using 121 as a witness. (Indicate all the intermediate steps.)
Hint: $121^{5564} \bmod 11131=92$
(b) Bob generates two primes, $p=113$ and $q=109$, to be used with the RSA algorithm.

From this determine $n, \varphi(n)$ and the decryption key $d$, given that $e=101$.
Hint:

$$
\begin{aligned}
& 101 \times d_{1}=78 \times Q+r_{1} \\
& 101 \times d_{2}=79 \times Q+r_{2} \\
& 101 \times d_{3}=80 \times Q+r_{3} \\
& 101 \times d_{4}=81 \times Q+r_{4} \\
& 101 \times d_{5}=82 \times Q+r_{5}
\end{aligned}
$$

(c) Encrypt the message 00054 .
(Hint: $54^{100} \equiv 5706$, using the specified modular arithmetic)

## Question 4

Consider the even parity code $C$ used on 8 -bit bytes, i.e., a codeword $c$ is in the form $\left(v_{0}, u_{0}, u_{1}, \ldots, u_{6}\right)$, where $v_{0}$ is the redundancy and $u_{i}, i \in\{0,1, \ldots, 6\}$ is the message. Also, $v_{0}$ is equal to 0 if the number of ones in the information part is even, else it is a logical one.
(a) Derive the parity-check equations for the code $C$.
( 2 marks)
(b) Determine the systematic generator matrix $G$ for the code $C$.
( 3 marks)
(c) Determine the parity-check matrix $H$ for the code $C$.
(d) Determine the minimum distance of the code $C$ and comment on the error detection and error correction capabilities of $C$.

## Question 5

Consider the polynomial $g(x)=1+x^{3}+x^{4}+x^{5}+x^{8}$.
(a) Show that $g(x)$ generates a code $C$ of length $n=17$.
(b) Determine the parameter $k$ of the code $C$.
( 2 marks)
(c) Systematically encode the message $(1,1, \ldots, 1)$.
( Total 10 marks)
( Test Total 55 marks)
$(100 \%=50$ marks $)$

