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University of the Witwatersrand, Johannesburg

Course or topic No(s) $\square$

Course or topic name(s)
Paper Number \& title

Examination/Test* to be
held during month(s) of
(*delete as applicable)
Year of Study
(Art \& Sciences leave blank)

Degrees/Diplomas for which
this course is prescribed
(BSc (Eng) should indicate which branch)

Faculty/ies presenting candidates

Internal examiners
and telephone
number(s)

External examiner(s)

Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)

Time allowance

Prof. T. G. Swart


| Course <br> Nos | ELEN3015 | Hours | 3 |
| :---: | :---: | :---: | :---: |


| Answer $A L L$ questions. |
| :---: |
| Closed book |
| Engineering calculator permitted |
| A4 handwritten information sheet |
| Total marks: 100 - Full marks: 100 |

Internal Examiners or Heads of School are requested to sign the declaration overleaf

1. As the Internal Examiner/Head of School, I certify that this question paper is in final form, as approved by the External Examiner, and is ready for reproduction.
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Name:
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Note: Show all workings, complete with the necessary comments. Marks will be allocated for all working and logical reasoning and not just for the correct answer.All terms and symbols are as defined in the course handouts. Answers written on your question paper will NOT be marked. Answers written in pencil will NOT be marked.

## Question 1

A columnar transposition cipher scheme is used for two parties who communicate securely over an open channel. The eavesdropper knows that the number of columns used in this cipher is not more than eight (8). The following ciphertext is captured by an eavesdropper:

## itaeini nratnim tetneex otdfope icrhxcu methrcr wseithe

(a) Show the method to cryptanalyze the ciphertext by using the bigram.
(b) Show the sums of frequency of 5 different possible solutions (widths).
(c) Show the most likely plaintext and number of letters in each cipher block. A noncontinuous plaintext could be obtained if the eavesdropped sequence is composed of two fractions from two consecutive cipher blocks.
$\left(\begin{array}{llllll}\text { th } & 1.52 \% & \text { en } & 0.55 \% & n g & 0.18 \% \\ h e & 1.28 \% & \text { ed } & 0.53 \% & \text { of } & 0.16 \% \\ \text { in } & 0.94 \% & \text { to } & 0.52 \% & \text { al } & 0.09 \% \\ e r & 0.94 \% & \text { it } & 0.50 \% & \text { de } & 0.09 \% \\ a n & 0.82 \% & \text { ou } & 0.50 \% & \text { se } & 0.08 \% \\ r e & 0.68 \% & \text { ea } & 0.47 \% & \text { le } & 0.08 \% \\ n d & 0.63 \% & \text { hi } & 0.46 \% & \text { sa } & 0.06 \% \\ a t & 0.59 \% & \text { is } & 0.46 \% & \text { si } & 0.05 \% \\ \text { on } & 0.57 \% & \text { or } & 0.43 \% & \text { ar } & 0.04 \% \\ n t & 0.56 \% & \text { ti } & 0.34 \% & \text { ve } & 0.04 \% \\ h a & 0.56 \% & \text { as } & 0.33 \% & \text { ra } & 0.04 \% \\ e s & 0.56 \% & \text { te } & 0.27 \% & \text { ld } & 0.02 \% \\ \text { st } & 0.55 \% & \text { et } & 0.19 \% & \text { ur } & 0.02 \%\end{array}\right)$

## Question 2

This question concerns the Discrete Cosine Transform (DCT) based lossy scheme which is part of the Joint Photographic Experts Group (JPEG) standard.
(a) Draw the block diagram of JPEG compression process.
(b) Explain why the above-mentioned compression process is a lossy process and why the lossy scheme is valid for image compression.
(c) In an application of steganography, one tries to store secret data in a JPEG image. Give a solution by using the standard JPEG compression process.

## Question 3

We consider the Galois field $\operatorname{GF}\left(2^{3}\right)$ based on the primitive polynomial $h(x)=1+x+x^{3}$.
(a) Derive the Galois field based on the given primitive polynomial in terms of binary sequences, polynomial notations and powers of the primitive element $(\alpha)$.
(b) Derive the corresponding minimum polynomials.
(c) Derive the generator polynomial of a single-error-correcting code based on the minimum polynomials. What is the rate of the code generated by the derived generator polynomial?

## Question 4

The frequency band between 100 kHz and 101 kHz is allocated to a communication system. The signal power is $S=31$ power unit per hertz. The noise in the band is additive white Gaussian noise with double-sided power spectral density $N_{0}=1 / 2$ power unit per hertz.
(a) What is the Shannon limit on the achievable data rate (bits/sec)?
( 5 marks)
(b) For a given bandwidth between 100 kHz and 110 kHz , and transmission data rate of $10^{5} \mathrm{bits} / \mathrm{sec}$, what is the required signal-to-noise ratio in decibels $(\mathrm{dB})$ ?
( 5 marks)
(Total 10 marks)

## Question 5

(a) A memoryless information source has a countably infinite symbol alphabet $\mathbf{S}=$ $\left\{S_{1}, S_{2}, \ldots\right\}$ with $P_{i}=a \alpha^{i}$ for $i=1,2, \ldots$ Express $a$ in terms of $\alpha$.
(b) Calculate the entropy of $\mathbf{S}$ as a function of $\alpha$.
( 5 marks)

Hint: $\sum_{i=1}^{\infty} a^{i}=\frac{a}{1-a}$ and $\sum_{i=1}^{\infty} i a^{i}=\frac{a}{(1-a)^{2}}$ given $|a| \leq 1$.
( Total 10 marks)

## Question 6

(a) Draw a data-flow diagram of the encryption and decryption process of a cipher block chain. What is the significance of using an initialization vector?
( 5 marks)
(b) Draw a data flow diagram to show the ciphertext stealing technique for a cipher block chain. What is the significance of using ciphertext stealing?

## Question 7

Let the irreducible polynomial for $\mathrm{GF}\left(2^{8}\right)$ be $m(x)=x^{8}+x^{4}+x^{3}+x+1$ (not primitive). The MixColumn Transformation is defined as

$$
M C=\left(\begin{array}{cccc}
\alpha & \alpha+1 & 1 & 1 \\
1 & \alpha & \alpha+1 & 1 \\
1 & 1 & \alpha & \alpha+1 \\
\alpha+1 & 1 & 1 & \alpha
\end{array}\right)\left(\begin{array}{cccc}
B_{0,0} & B_{0,1} & B_{0,2} & B_{0,3} \\
B_{1,1} & B_{1,2} & B_{1,3} & B_{1,0} \\
B_{2,2} & B_{2,3} & B_{2,0} & B_{2,1} \\
B_{3,3} & B_{3,0} & B_{3,1} & B_{3,2}
\end{array}\right)
$$

Given $B_{0,0}=89_{16}, B_{1,1}=0_{16}, B_{2,2}=A B_{16}$ and $B_{3,3}=C D_{16}$, calculate the four elements in the first column of the resultant matrix.

## Question 8

Consider a $(7,4)$ Hamming code with parity-check matrix,

$$
H=\left(\begin{array}{lllllll}
1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1
\end{array}\right)
$$

Add one more parity check bit with the binary-sum value of all the existing 7 bits.
(a) What is the new parity-check matrix?
(b) What is the minimum Hamming distance of the new code? Prove it.
(c) If the received sequence is $[00101 \mathrm{E} 000$ (E denotes erasure error), determine the sent codeword using the syndrome decoding algorithm.

