Tutorial 8: Information Theory

1. Search the Internet for information of different source models of information. Specifically look for:
   a. Zero-memory sources
   b. Markov Sources of various orders
   c. Adjoint Sources

2. Consider a source of information that produces the following sequence of messages: $M = M_1M_2M_1M_3M_4$. Consider the following about the messages:

Message Number	Probability
$M_1 = 00$	½
$M_2 = 01$	$^1/_6$
$M_3 = 10$	$^1/_6$
$M_4 = 11$	$^1/_6$

   a. Given the above, determine the entropy H of the message M.
   b. What is the average information content per bit in any of the messages M_i?

3. You may wish to use the following identities in answering this question

$$\sum_{1}^{\infty} \alpha^n = \frac{\alpha}{1-\alpha} \quad \text{and} \quad \sum_{1}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \quad \alpha \approx 0 \qquad \text{for } 0 \le \alpha < 1$$

   a. A zero-memory information source has a countably infinite symbol alphabet $S = \{S_1, S_2, \ldots\}$ with $P_i = a\alpha^i \ \forall i$. Express a in terms of α.
   b. Find and sketch $H(S)$ as a function of α. Note particularly the behaviour of H(S) for $\alpha \approx 0$ and $\alpha \approx 1$.

4. Assuming that the probability of binary symbols in the following sample is representative of the source that produced them:
   a. Calculate the entropy $H(M)$ of the source.

   `0110011111100110110101111 0`

   b. Given that for any binary source, the "compression factor" is given by $\frac{1}{H(M)}$. Calculate the expected compression factor attainable using a statistical compression algorithm to compress data produced by this source.

5. Using the letter frequency distribution of English, given in a table in the beginning of the course, calculate the entropy of the English language.